# Overview of Soil-Structure Interaction Principles

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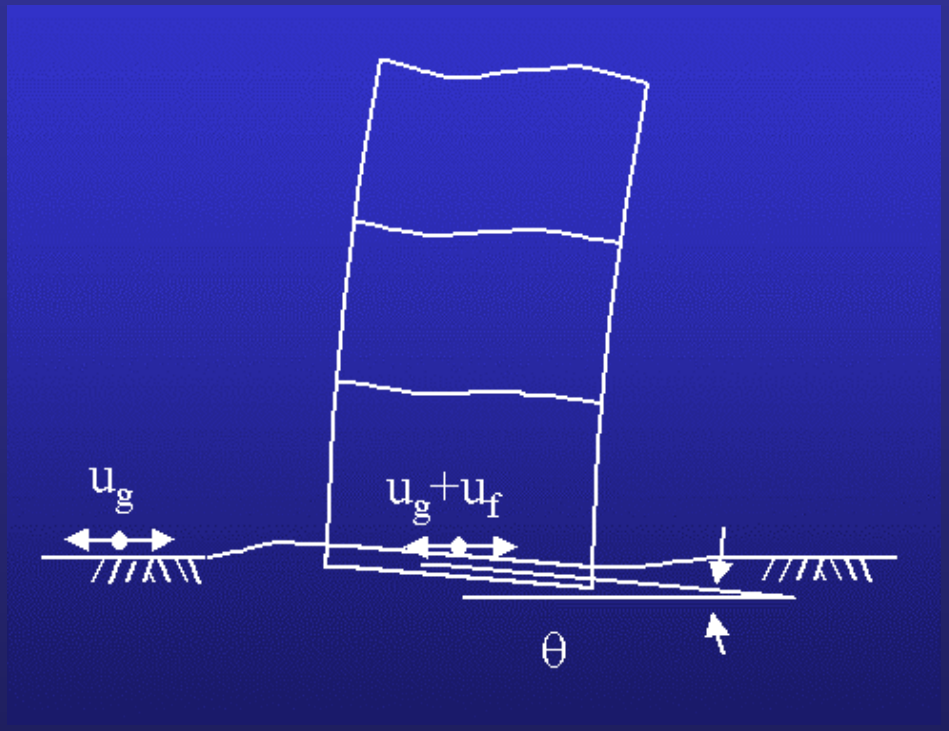
# Overview

A. Introduction
B. General methods of analysis
C. Inertial interaction
D. Kinematic interaction

# A. Introduction

Response dictated by interactions between:

- Structure
- Foundation
- Underlying soil/rock



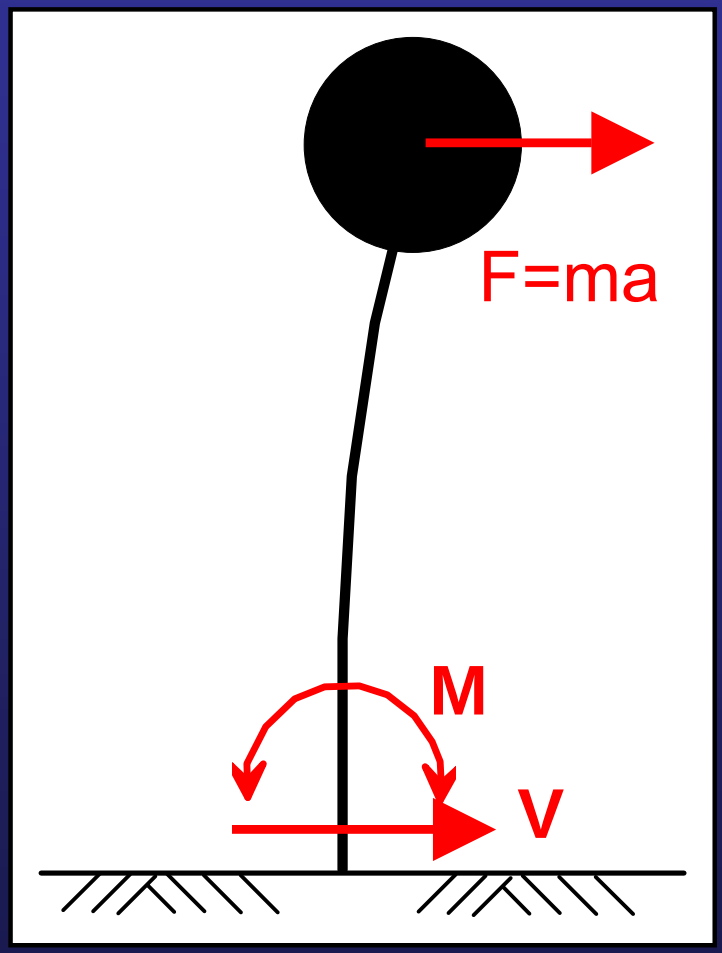

System analysis evaluates response given free-field motion, $u_g$

No SSI when_____

SSI effect =_____

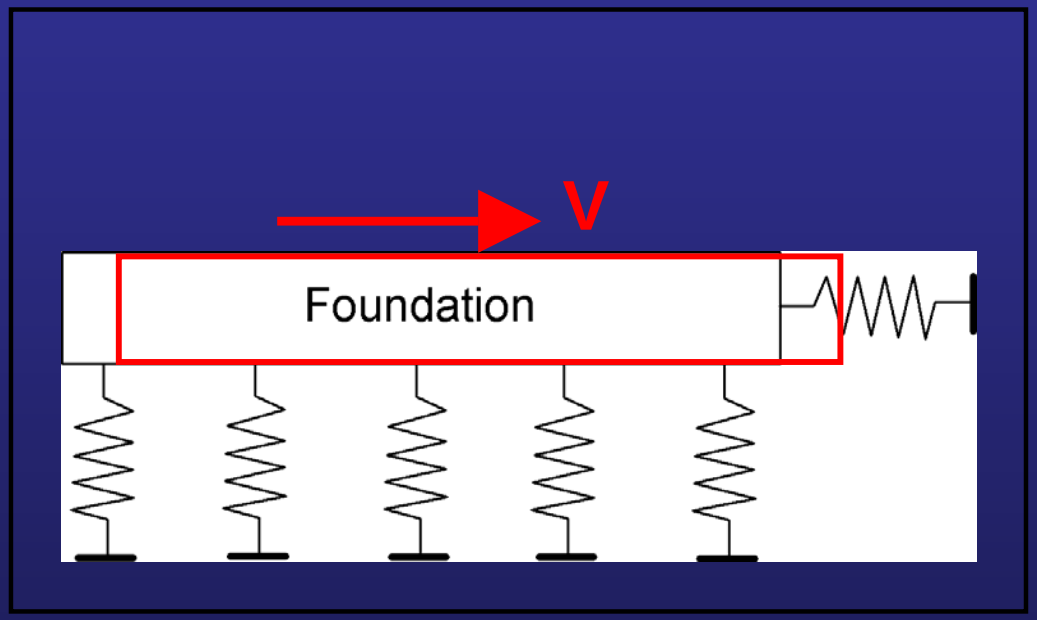
# A. Introduction.
# Three critical aspects of SSI

- Inertia $\rightarrow$ base shear (V) and moment (M)



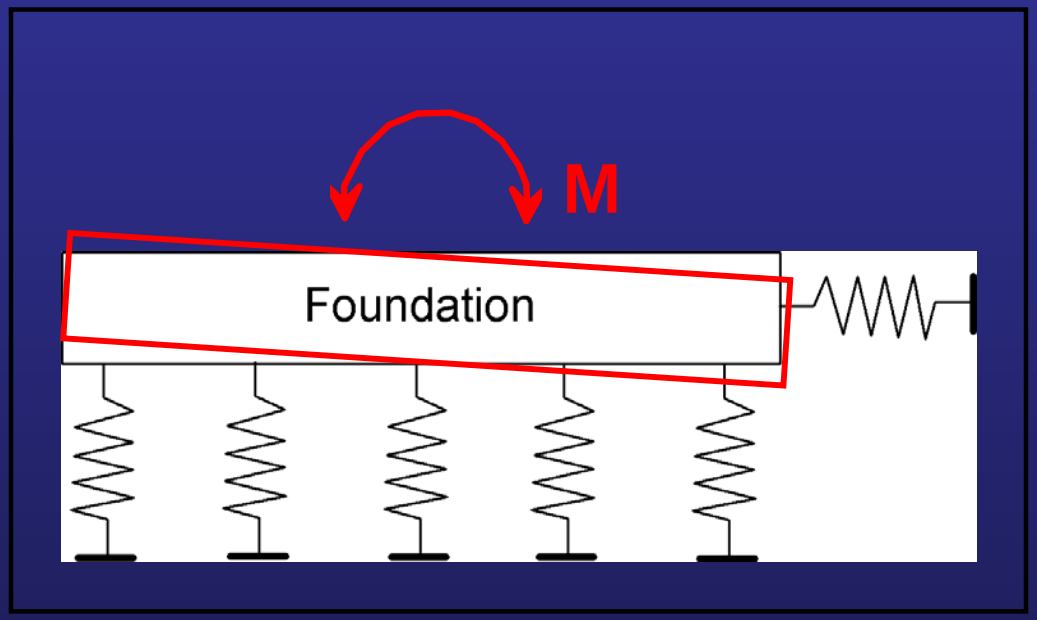
# A. Introduction.
# Three critical aspects of SSI

- Inertia $\rightarrow$ base shear (V) and moment (M)
- V $\rightarrow$ relative foundation/free-field displacement ($u_f$)



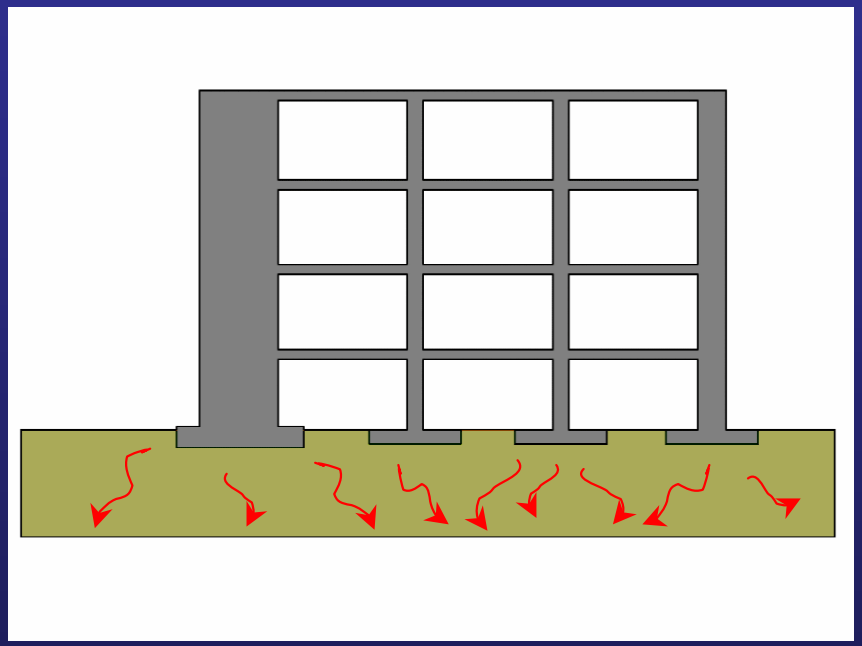
# A. Introduction.
# Three critical aspects of SSI

- Inertia $\rightarrow$ base shear (V) and moment (M)
- V $\rightarrow$ relative foundation/free-field displacement ($u_f$)
- M $\rightarrow$ relative foundation/free-field rotation ($\theta_f$)



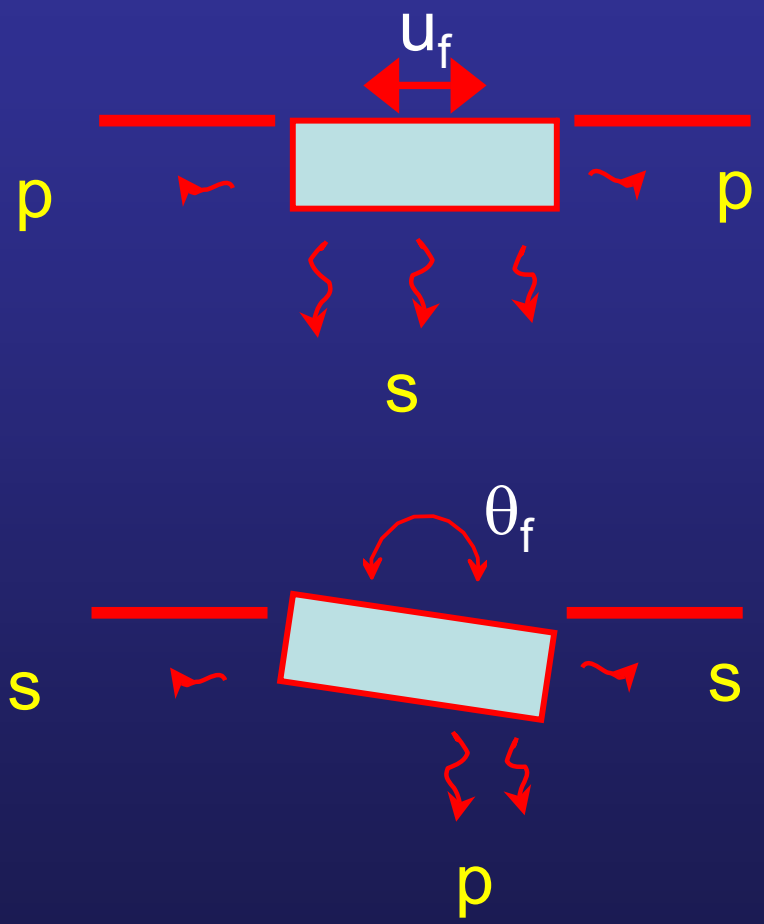
# A. Introduction.
# Three critical aspects of SSI

- $u_f$, $\theta_f \rightarrow$ foundation damping



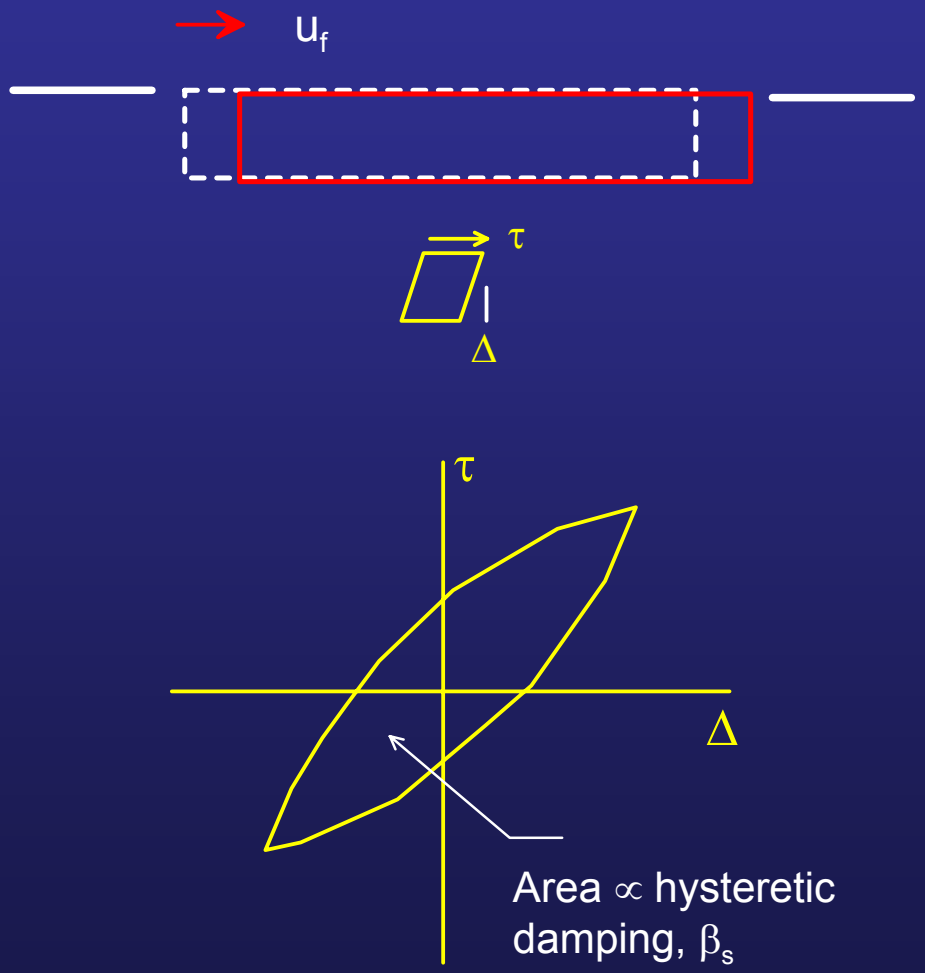
# A. Introduction. Three critical aspects of SSI

- $u_f$, $\theta_f \rightarrow$ foundation damping
- Radiation damping – foundation acts as wave source



# A. Introduction. Three critical aspects of SSI

- $u_f$, $\theta_f \rightarrow$ foundation damping
- Radiation damping – foundation acts as wave source
- Hysteretic damping in soil



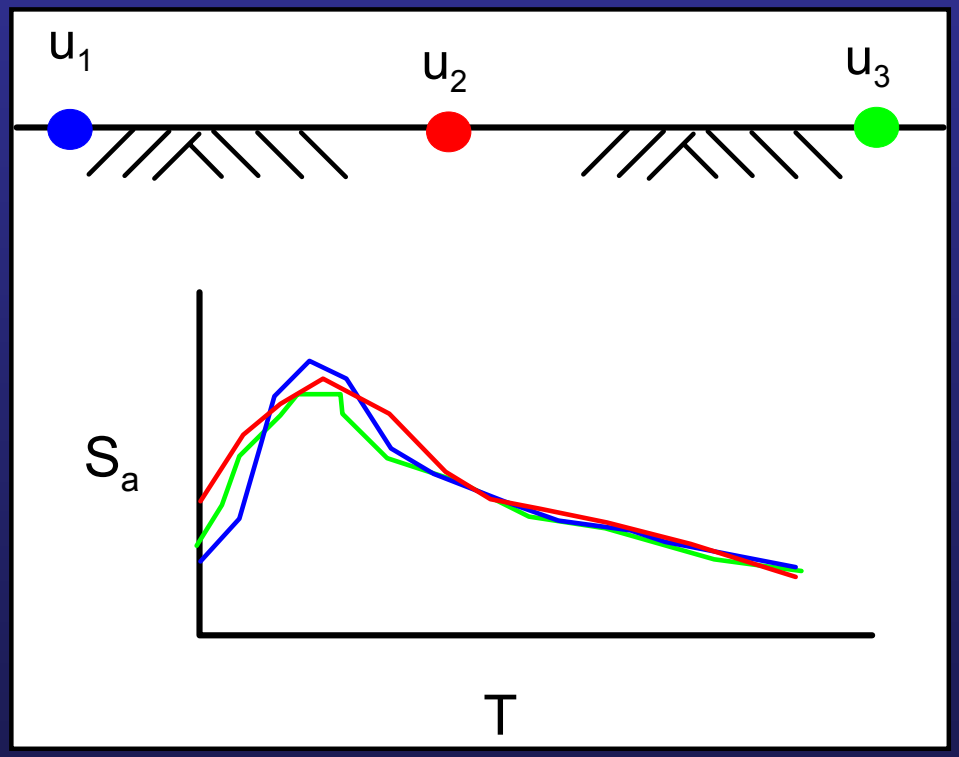
# A. Introduction.
# Three critical aspects of SSI

1. Inertial soil structure interaction
   - Inertia from vibration of structure and foundation
   - Causes foundation translation and rotation ($u_f$ and $\theta_f$)
     - Directly affects system flexibility and mode shapes
     - Introduces foundation damping

# A. Introduction.
# Three critical aspects of SSI

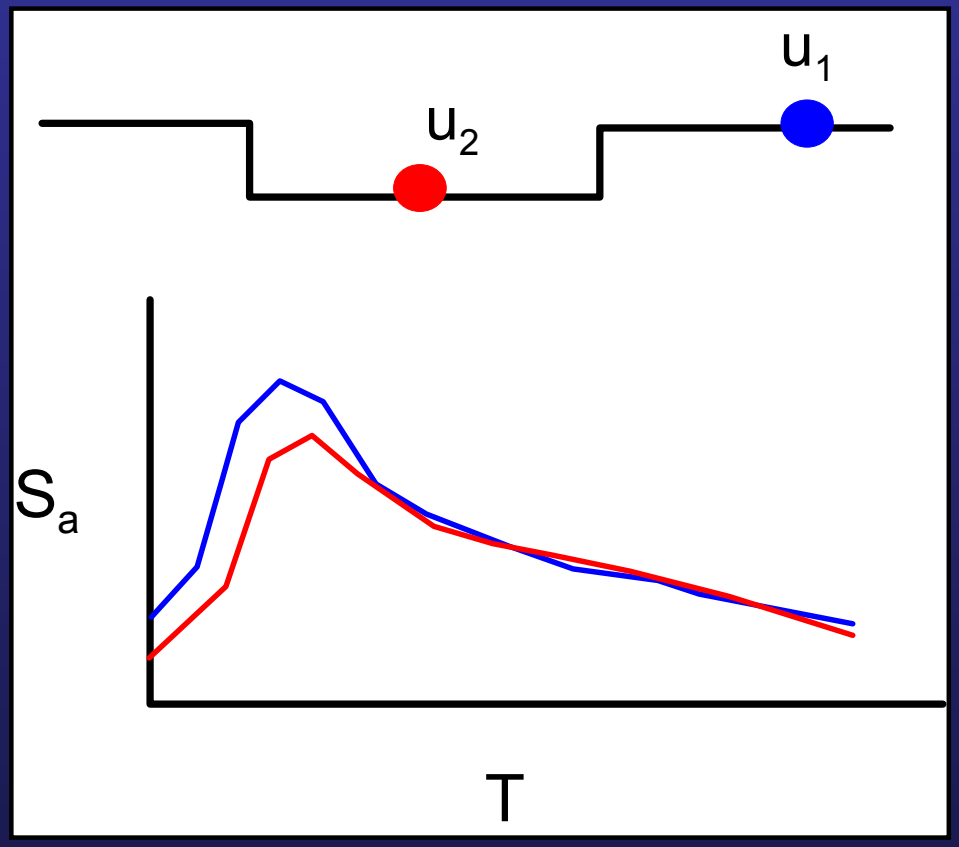
2. Kinematic interaction

- Incoherent ground motions $\rightarrow$ base slab averaging



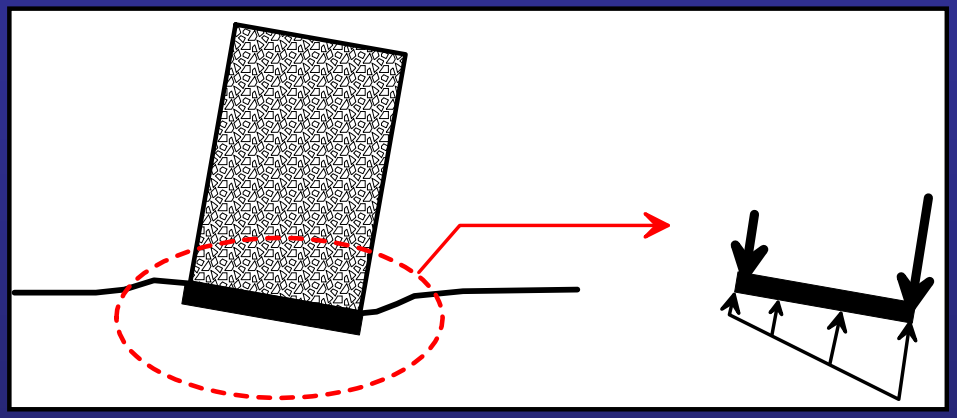
# A. Introduction.
# Three critical aspects of SSI

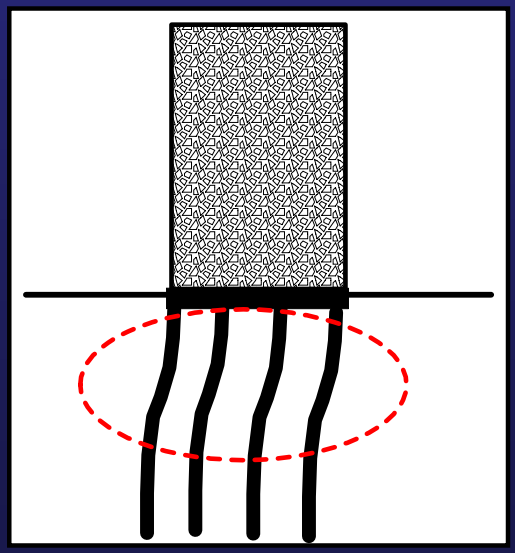
2. Kinematic interaction
   - Incoherent ground motions → base slab averaging
   - Ground motion reductions with depth



# A. Introduction.
# Three critical aspects of SSI

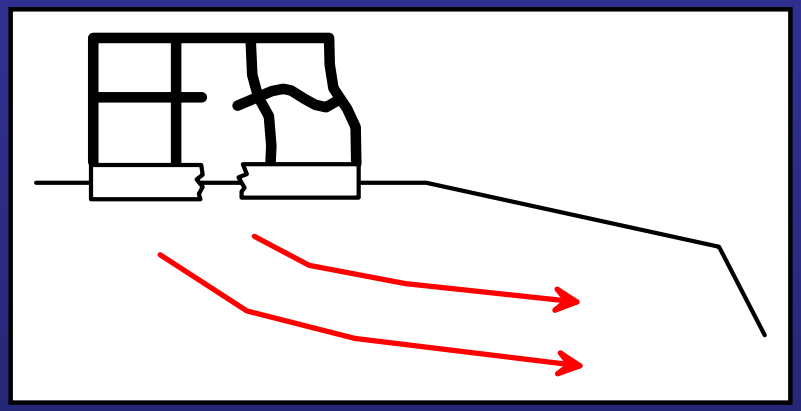
3. Foundation deformations
   - Loads from superstructure inertia

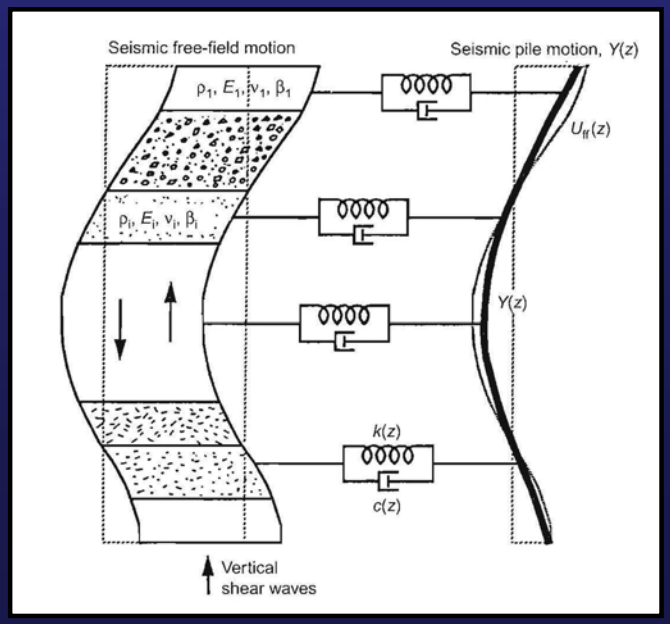




# A. Introduction.
# Three critical aspects of SSI

3. Foundation deformations
   - Loads from superstructure inertia
   - Deformations applied by soil






Nikolaou et al. (2001)

*Beyond scope of current presentation*

# B. General Methods of Analysis

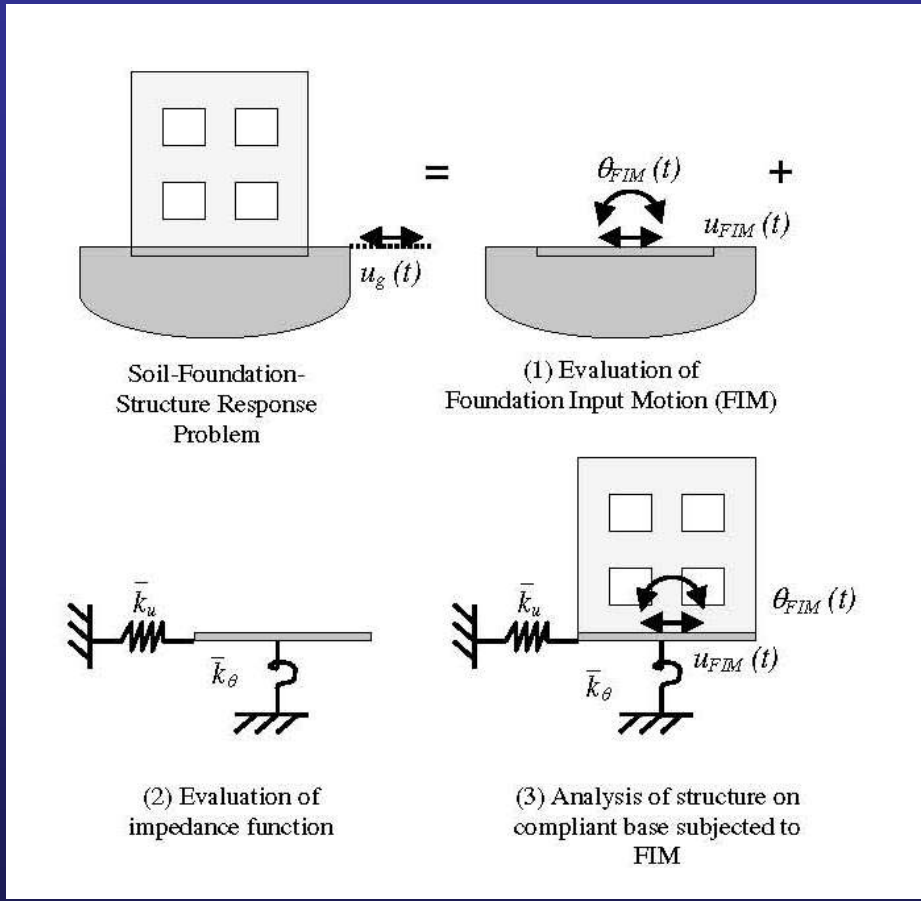
- Direct approach
  - Full modeling of soil, foundation, structure
  - Propagate waves through system

*Beyond scope of current presentation*

# B. General Methods of Analysis

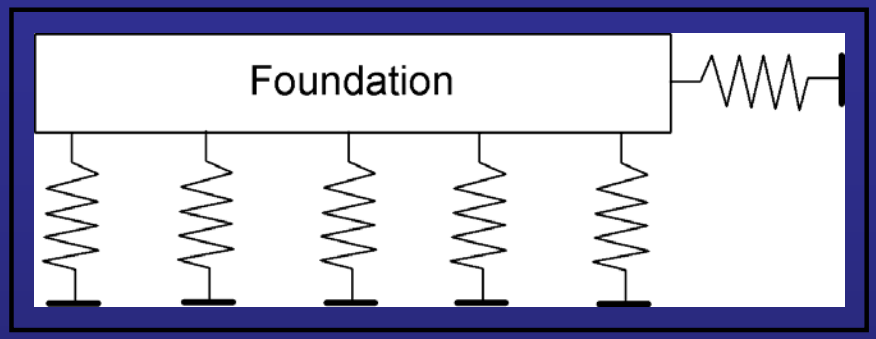
- Direct approach
- Substructure approach

***Focus of this seminar***



# C. Inertial Interaction

- Springs used to represent soil-foundation interaction
- Complex-valued
  - Real part represents stiffness
  - Imaginary part related to damping

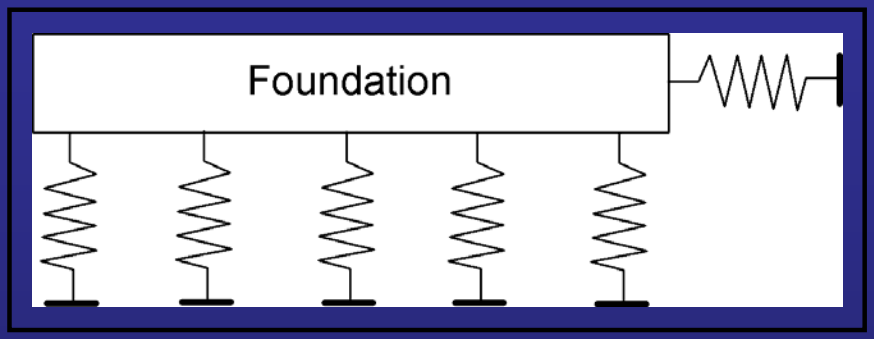


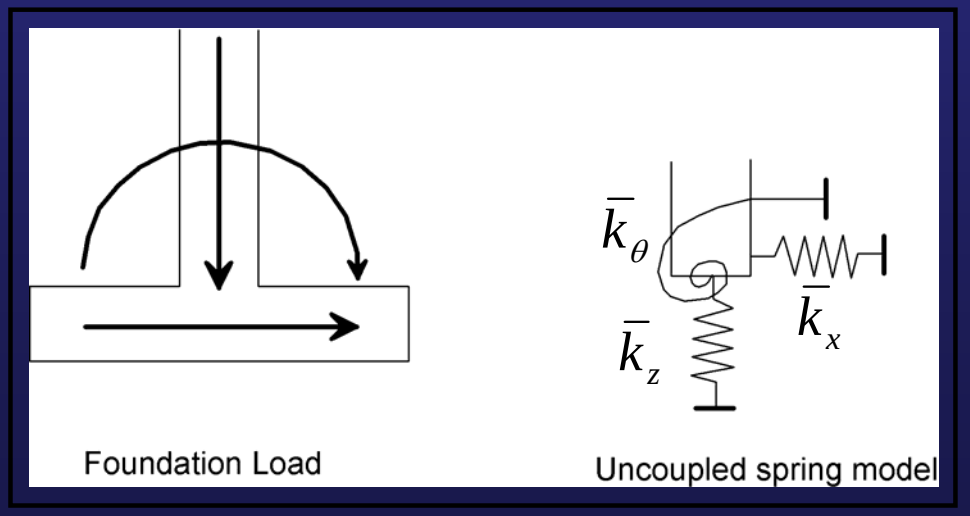

***Combination of real and complex parts comprises "Impedance function"***

# C. Inertial Interaction

- Springs used to represent soil-foundation interaction
- Complex-valued
- If rigid foundation, simplifies to:
  - 3 springs for 2D system
  - 6 springs for 3D system

$$\bar{k}_j = k_j(a_0, \upsilon) + i\omega c_j(a_0, \upsilon)$$



# C. Inertial Interaction. Effects on System Behavior

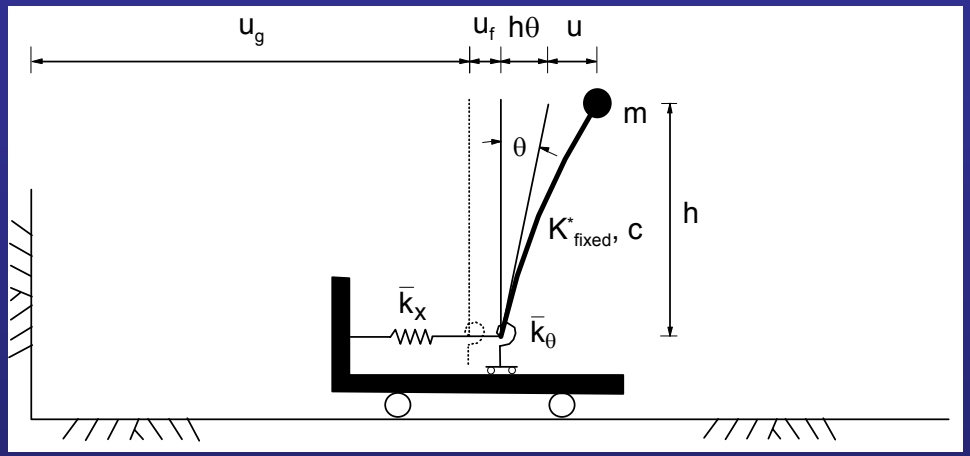
- Concepts of period lengthening and foundation damping
  - System period

$$\frac{\tilde{T}}{T} = \sqrt{1 + \frac{k_{fixed}}{k_x} + \frac{k_{fixed} h^2}{k_\theta}}$$

  - System damping

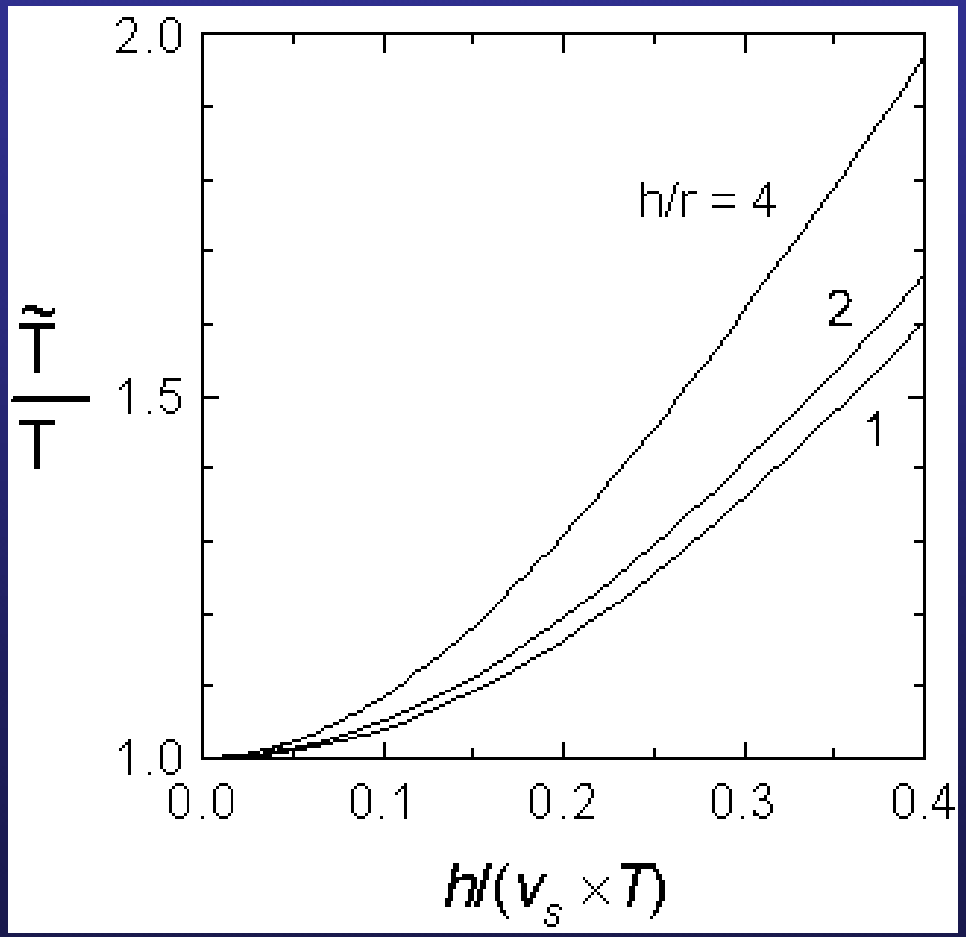
$$\beta_0 \doteq \beta_f + \frac{\beta_i}{\left(\tilde{T}/T\right)^3}$$

*Foundation damping factor*



# C. Inertial Interaction. Effects on System Behavior

- Concepts of period lengthening and foundation damping
- $\tilde{T}/T$, $\beta_f$ affected by:
  - $h/(v_s T)$ = ratio of str to soil stiffness
  - $h/r$ = str aspect ratio
  - $\gamma = m/(\rho \pi r^2 h)$ = ratio str. to soil mass (~0.1-0.2)



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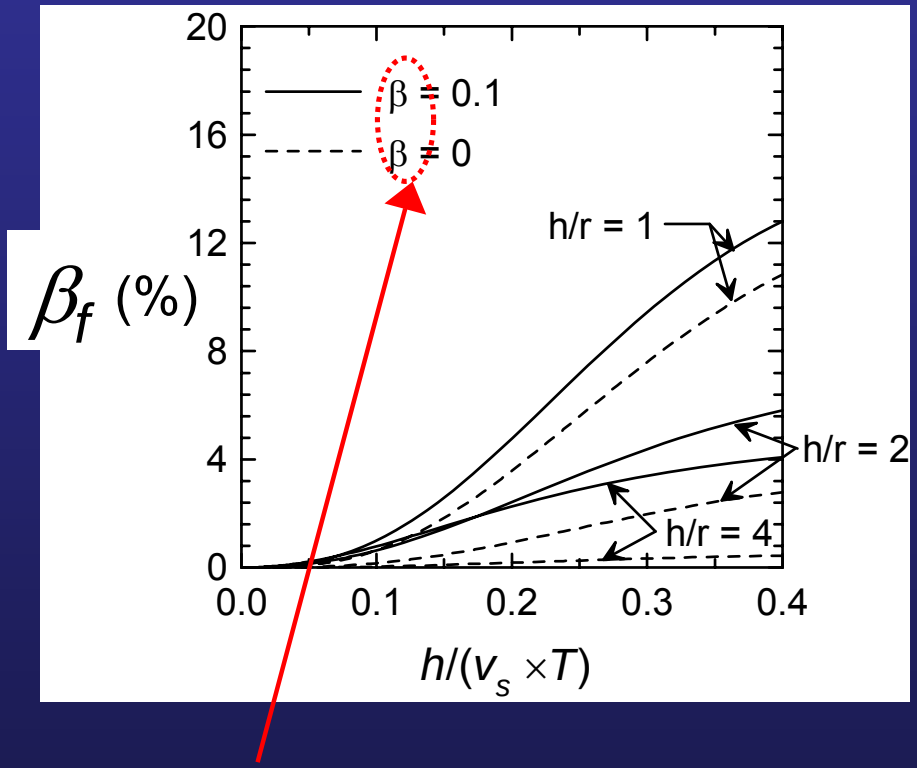
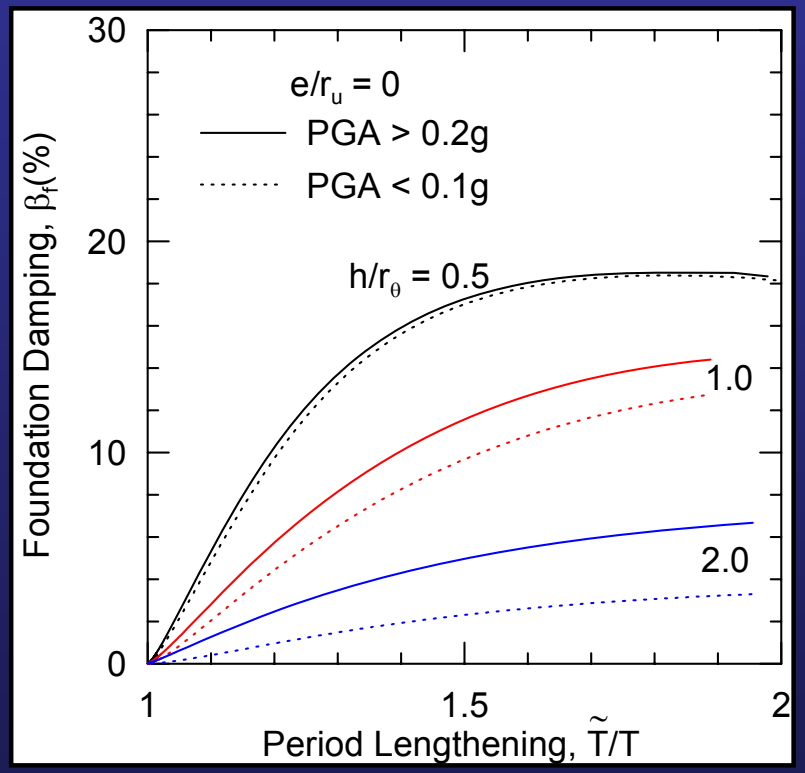
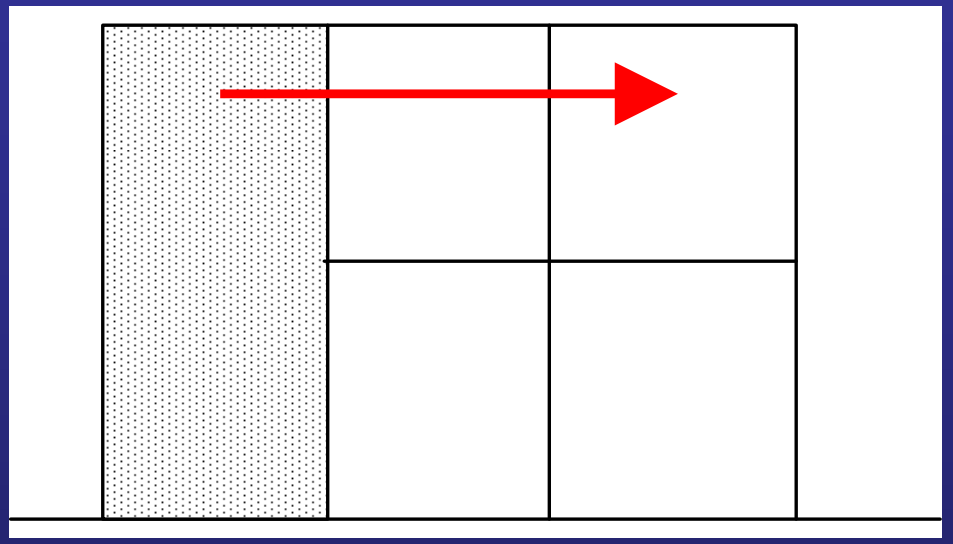


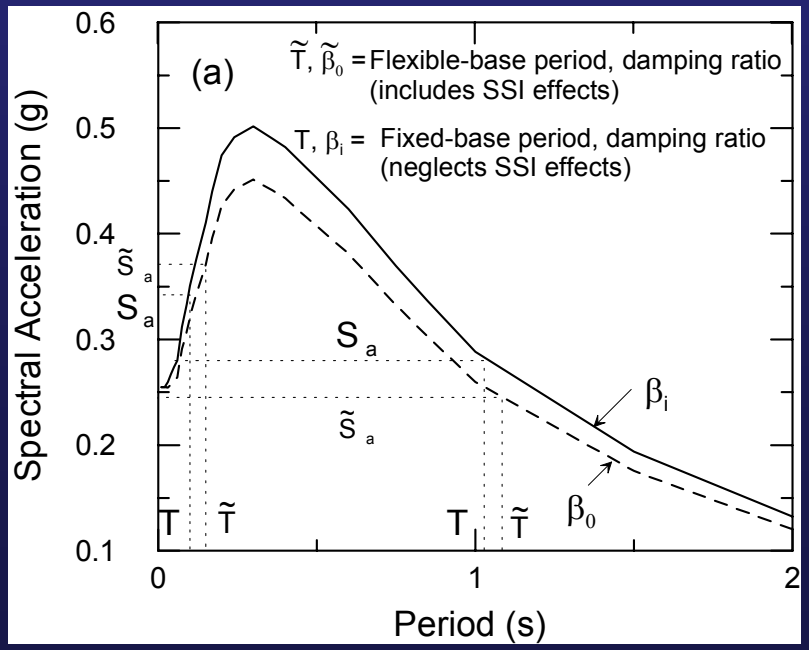

Hysteretic soil damping



# C. Inertial Interaction. Effects on Base Shear

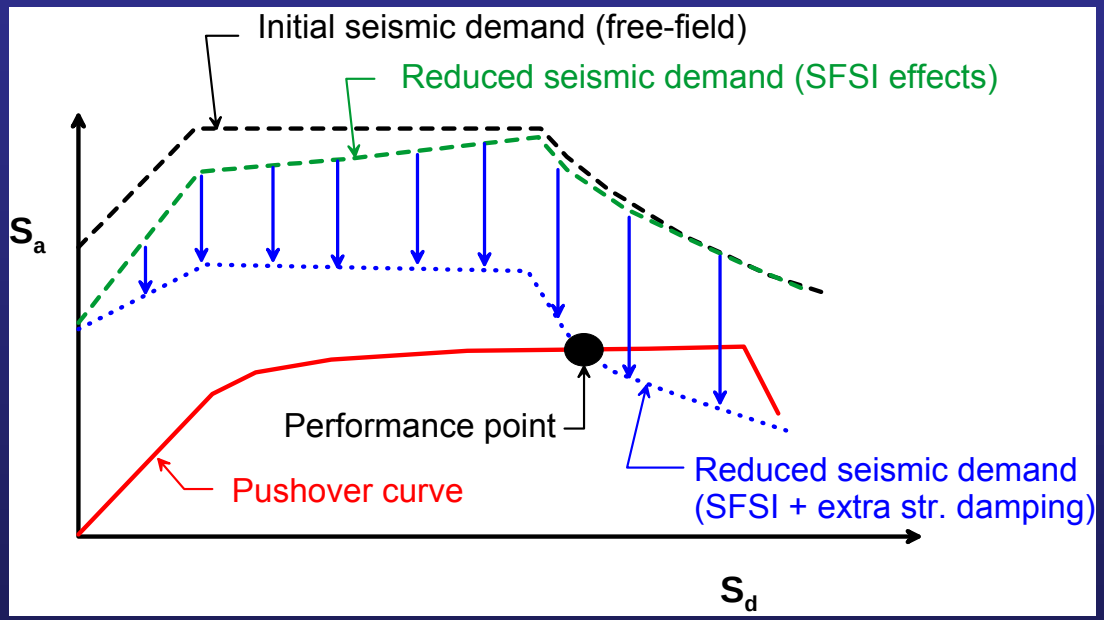
- Force-based procedure
- SSI affects design spectral ordinate
- Usually not considered for design of new buildings





# C. Inertial Interaction. Effects on Displacement-Based Pushover Analysis

- Initial seismic demand
  - Should be drawn for foundation motion, not free-field
  - Spectral ordinates should reflect system damping ratio
- Pushover curve
  - Soil springs in pushover analysis

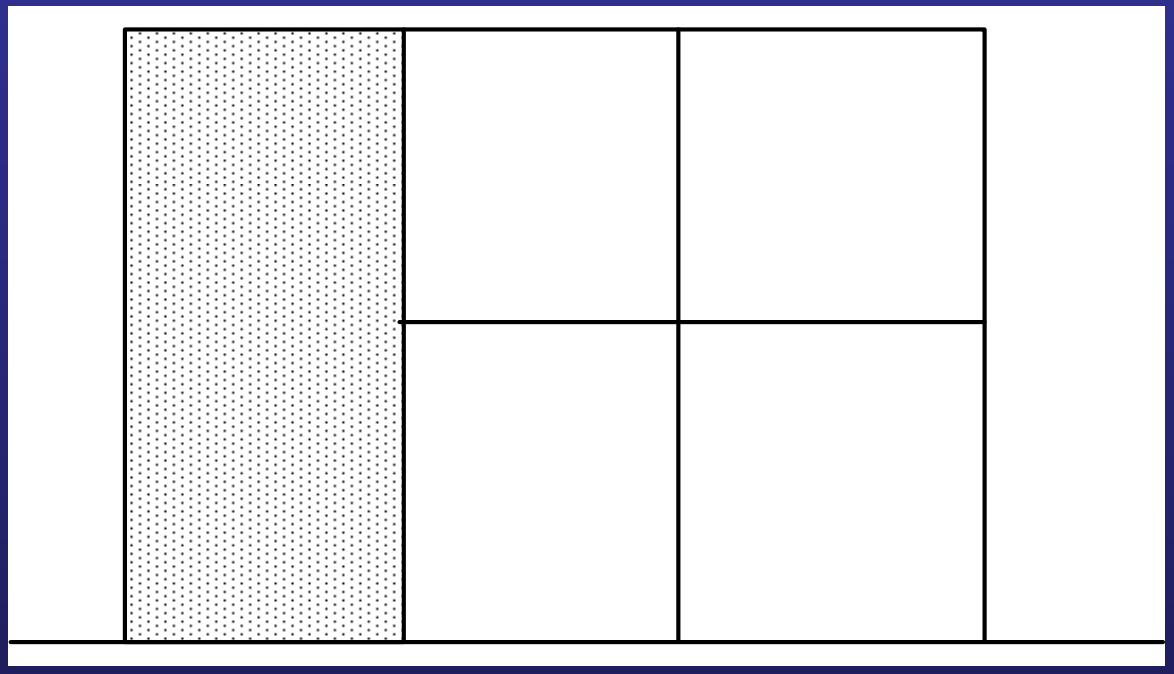


# Are these effects important?

- YES, especially for short-period structures
- Field data shows:
  - Foundation damping ratios up to ~ 10-20%
  - Period lengthening up to ~ 1.5
  - Foundation/ff Sa's at low period as low as ~0.5



# SSI Can Affect Retrofit Decisions



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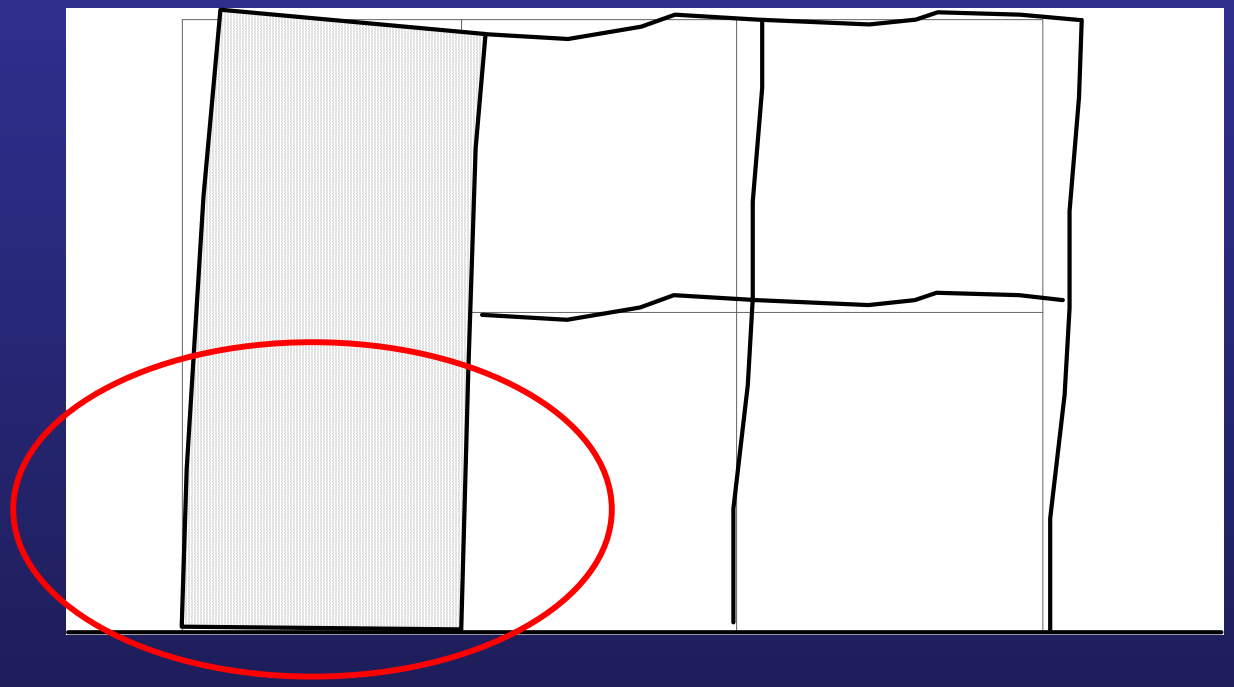
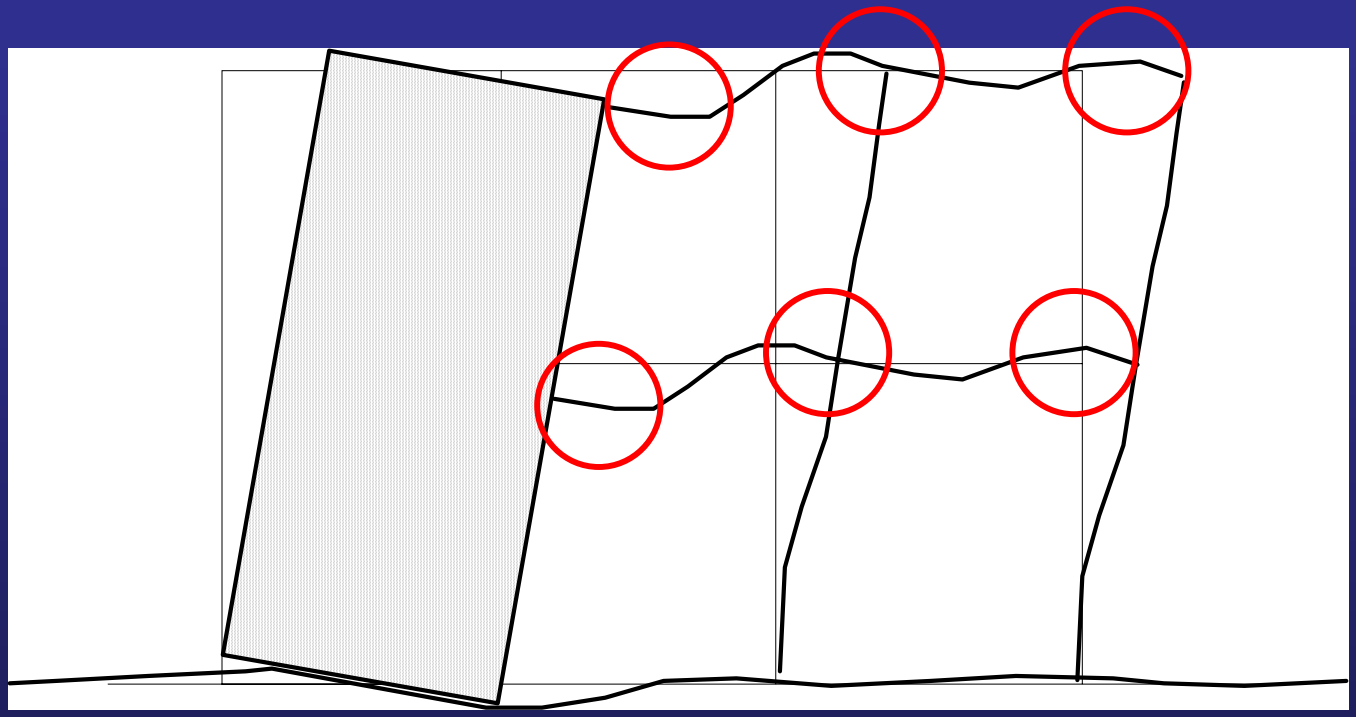


Fixed-Base

# SSI Can Affect Retrofit Decisions



Flexible-Base

# C. Inertial Interaction. Impedance Functions

$$\bar{k}_j = k_j(a_0, \upsilon) + i\omega c_j(a_0, \upsilon)$$

$a_0 = \omega r/V_s$ $\nu$ = Poisson's ratio

j = u (translation, x or z)
$\theta$ (rocking)

$$k_u = \alpha_u K_u \qquad c_u = \beta_u \frac{K_u r_u}{V_S} \qquad r_u = \sqrt{A_f/\pi}$$

$$k_\theta = \alpha_\theta K_\theta \qquad c_\theta = \beta_\theta \frac{K_\theta r_\theta}{V_S} \qquad r_\theta = \sqrt[4]{4I_f/\pi}$$

Two aspects of impedance function analysis:
1) Static stiffness (e.g., $K_x$)
2) Dynamic modifiers (e.g., $\alpha_x$, $\beta_x$)

# C. Inertial Interaction. Impedance Functions

## Static Stiffness (surface foundation)

Circle:

$$K_x = \frac{8}{2-\upsilon} G r_u$$

$$K_\theta = \frac{8}{3(1-\upsilon)} G r_\theta^3$$

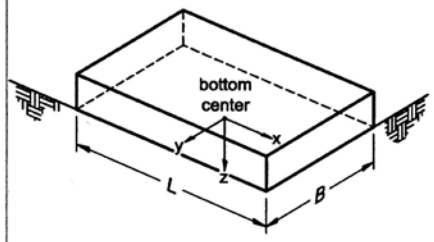
$$K_z = \frac{4}{1-\upsilon} G r_u$$

**Used in NEHRP Provisions**

Rectangle:

| Degree of freedom | Stiffness of foundation at surface |
|---|---|
| Translation along x-axis | $K_{x,sur} = \frac{GB}{2-\nu}\left[3.4\left(\frac{L}{B}\right)^{0.65} + 1.2\right]$ |
| Translation along y-axis | $K_{y,sur} = \frac{GB}{2-\nu}\left[3.4\left(\frac{L}{B}\right)^{0.65} + 0.4\frac{L}{B} + 0.8\right]$ |
| Translation along z-axis | $K_{z,sur} = \frac{GB}{1-\nu}\left[1.55\left(\frac{L}{B}\right)^{0.75} + 0.8\right]$ |
| Rocking about x-axis | $K_{xx,sur} = \frac{GB^3}{1-\nu}\left[0.4\left(\frac{L}{B}\right) + 0.1\right]$ |
| Rocking about y-axis | $K_{yy,sur} = \frac{GB^3}{1-\nu}\left[0.47\left(\frac{L}{B}\right)^{2.4} + 0.034\right]$ |
| Torsion about z-axis | $K_{zz,sur} = GB^3\left[0.53\left(\frac{L}{B}\right)^{2.45} + 0.51\right]$ |

Note:




Orient axes such that $L \geq B$

FEMA-356

# C. Inertial Interaction. Impedance Functions

## Static Stiffness (embedment modification)

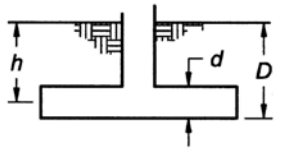
Circle:

$$(K_U)_E = K_U\left(1+\frac{2}{3}\frac{e}{r_u}\right) \qquad (K_\theta)_E = K_\theta\left(1+2\frac{e}{r_\theta}\right)$$

Rectangle:

| Degree of freedom | Correction factor for embedment |
|---|---|
| Translation along x-axis | $\beta_x = \left(1+0.21\sqrt{\frac{D}{B}}\right)\cdot\left[1+1.6\left(\frac{hd(B+L)}{BL^2}\right)^{0.4}\right]$ |
| Translation along y-axis | $\beta_y = \beta_x$ |
| Translation along z-axis | $\beta_z = \left[1+\frac{1}{21}\frac{D}{B}\left(2+2.6\frac{B}{L}\right)\right]\cdot\left[1+0.32\left(\frac{d(B+L)}{BL}\right)^{2/3}\right]$ |
| Rocking about x-axis | $\beta_{xx} = 1+2.5\frac{d}{B}\left[1+\frac{2d}{B}\left(\frac{d}{D}\right)^{-0.2}\sqrt{\frac{B}{L}}\right]$ |
| Rocking about y-axis | $\beta_{yy} = 1+1.4\left(\frac{d}{L}\right)^{0.6}\left[1.5+3.7\left(\frac{d}{L}\right)^{1.9}\left(\frac{d}{D}\right)^{-0.6}\right]$ |
| Torsion about z-axis | $\beta_{zz} = 1+2.6\left(1+\frac{B}{L}\right)\left(\frac{d}{B}\right)^{0.9}$ |

Note:




$d$ = height of effective sidewall contact (may be less than total foundation height)
$h$ = depth to centroid of effective sidewall contact

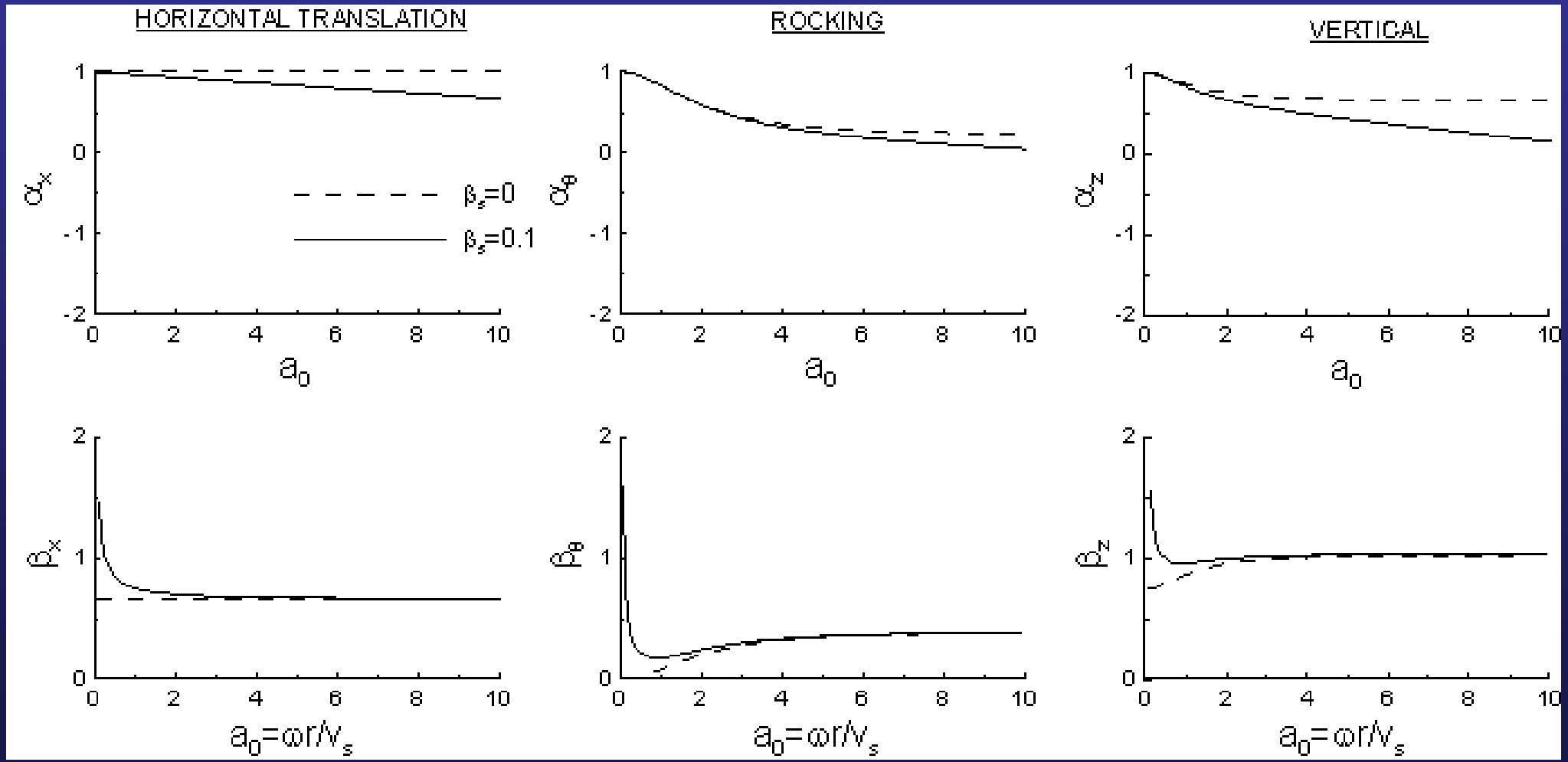
For each degree of freedom, calculate $K_{emb} = \beta K_{sur}$

**Figure 4-2** Elastic Solutions for Rigid Footing Spring Constants

**FEMA-356**

# C. Inertial Interaction. Impedance Functions

## Dynamic Modifiers (surface foundation)

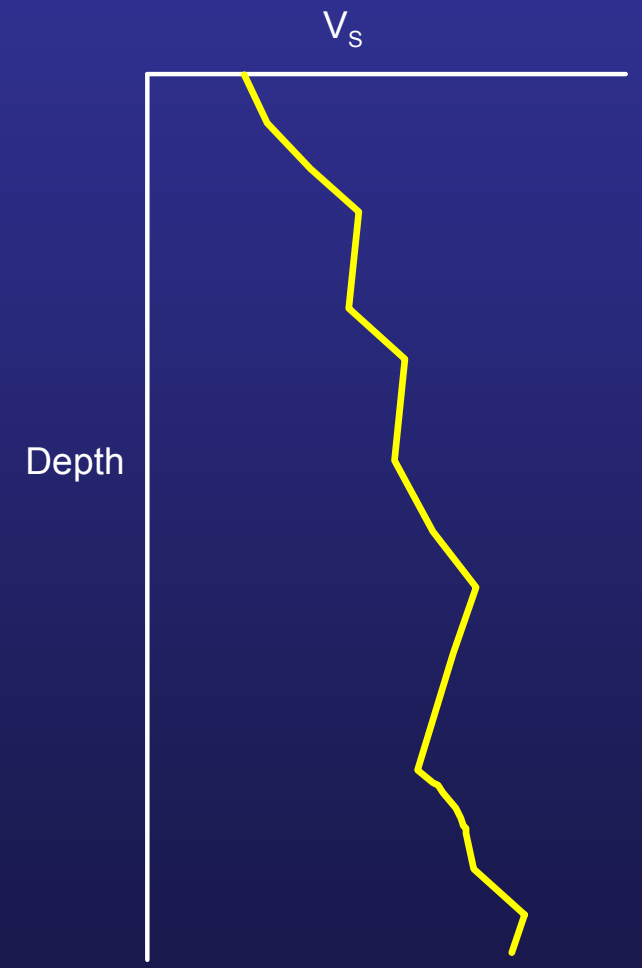



after Veletsos and Verbic, 1973

# C. Inertial Interaction. Impedance Functions

## Dealing with nonuniform profiles

- Issue: What is the effective $V_s$ for a non-uniform profile?
- $V_s$ increase with depth
  - Increases foundation stiffness
  - Impedes radiation damping at large $\lambda$ (low $f$) relative to halfspace



# C. Inertial Interaction. Impedance Functions

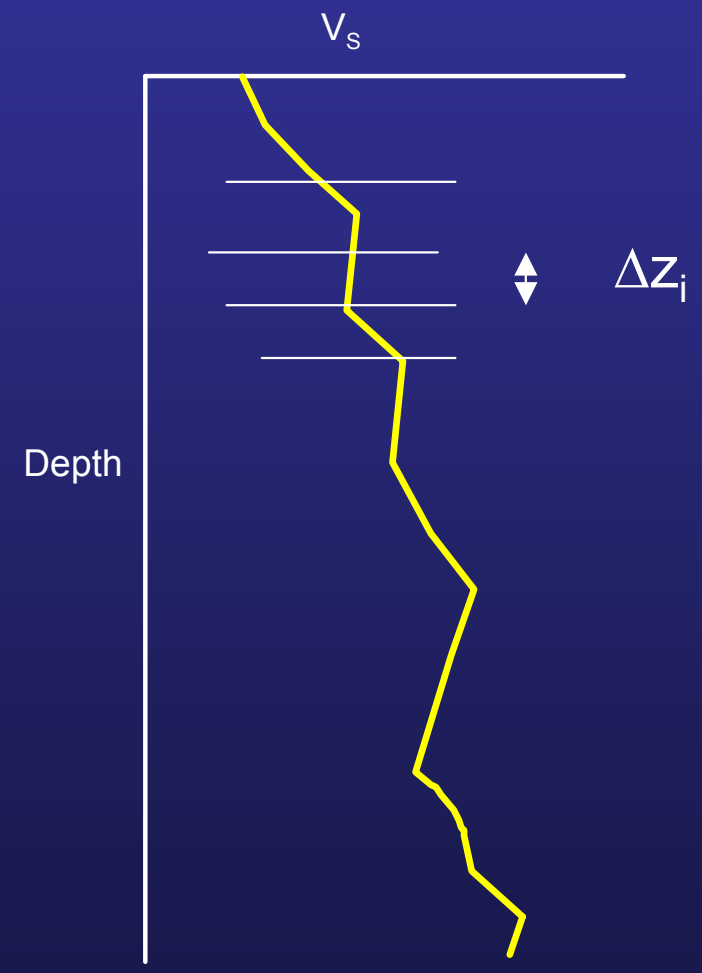
## Dealing with nonuniform profiles

- For stiffness, use

$$V_s = \frac{Z_e}{tt}$$

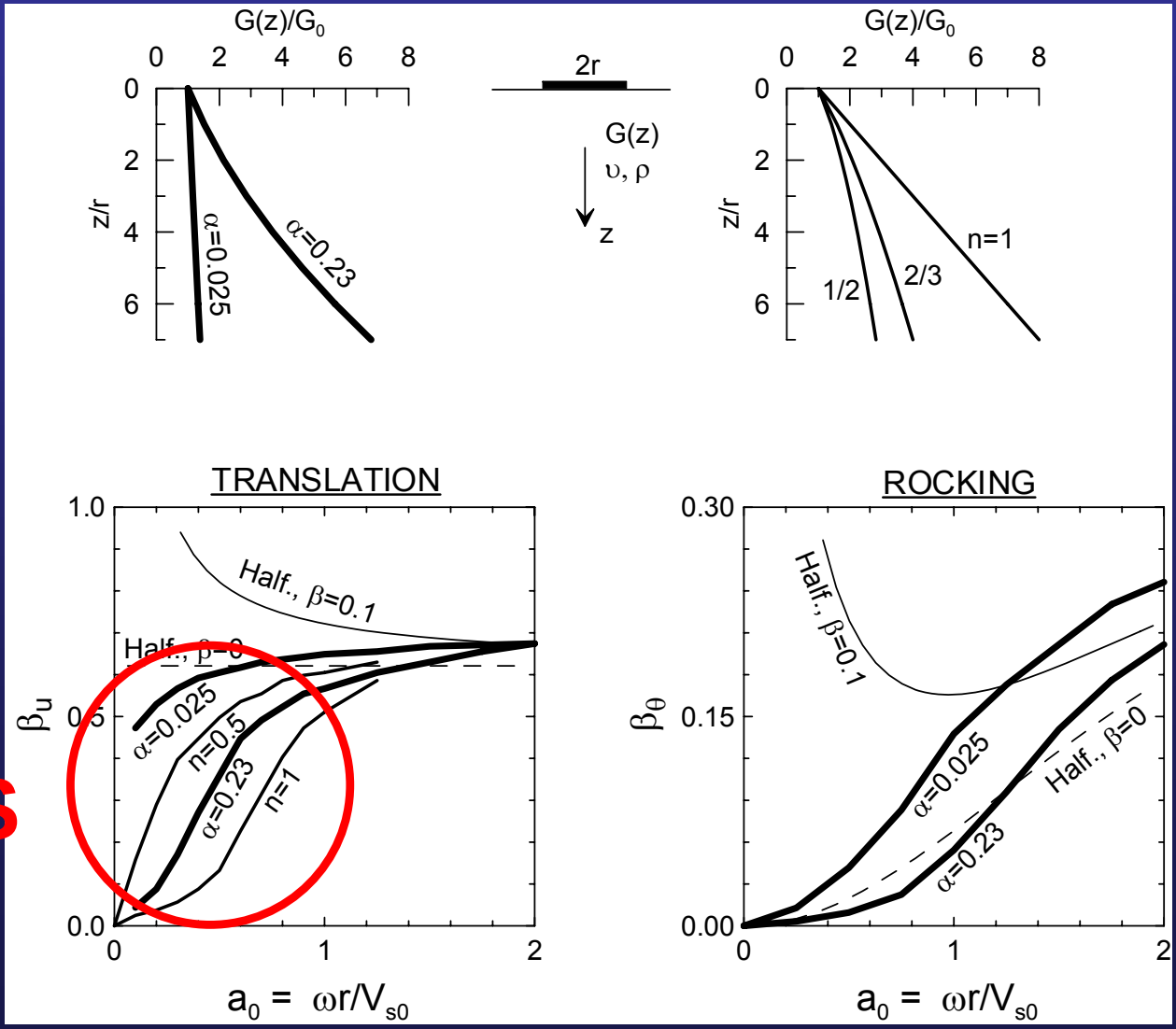
  - $Z_e = 0.75 r_u$ or $0.75 r_\theta$
  - $$tt = \sum \frac{\Delta z_i}{(V_s)_i}$$

- For damping, use $(V_s)_0$



# C. Inertial Interaction. Impedance Functions

## Dealing with nonuniform profiles




**BIAS**

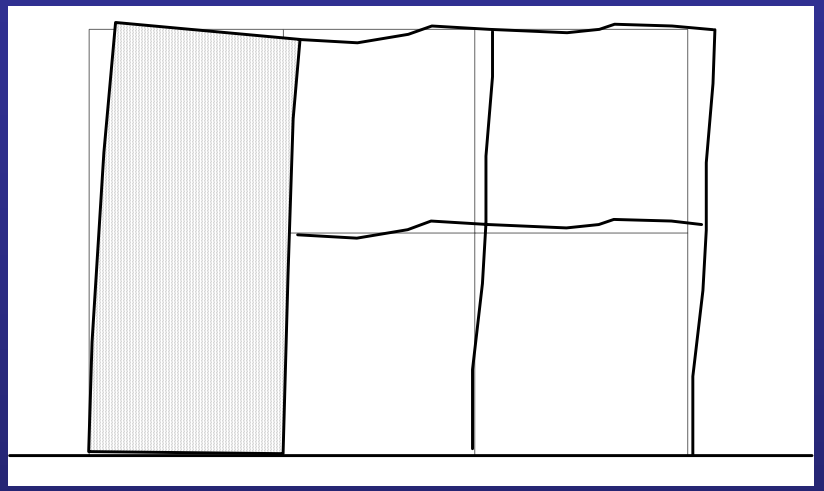
after Gazetas, 1991

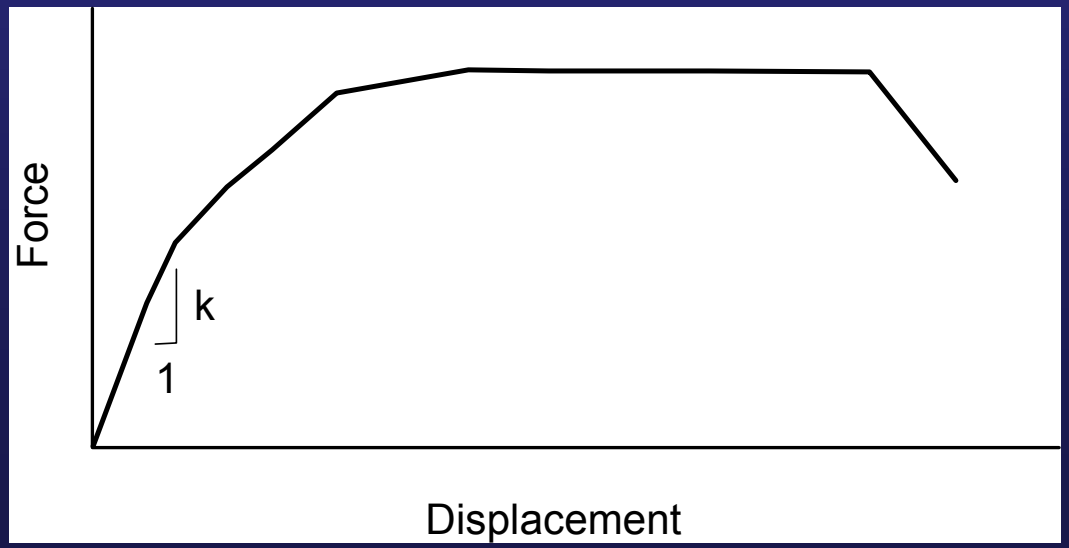
# C. Inertial Interaction. Typical Application

1. Evaluate foundation radii
   - $r_u = \sqrt{A_f / \pi}$
   - $r_\theta = \sqrt[4]{4 I_f / \pi}$
   - Analysis of $I_f$ must consider shear wall configuration and potential rotational coupling between walls
2. Evaluate foundation embedment, *e*
3. Evaluate effective height of structure, *h*
4. Initial fixed base damping, $\beta_i$ (usually 5%)

# C. Inertial Interaction. Typical Application

5. Evaluate $\tilde{T}/T$ using structure-specific model :
   - Fixed-base period $T$

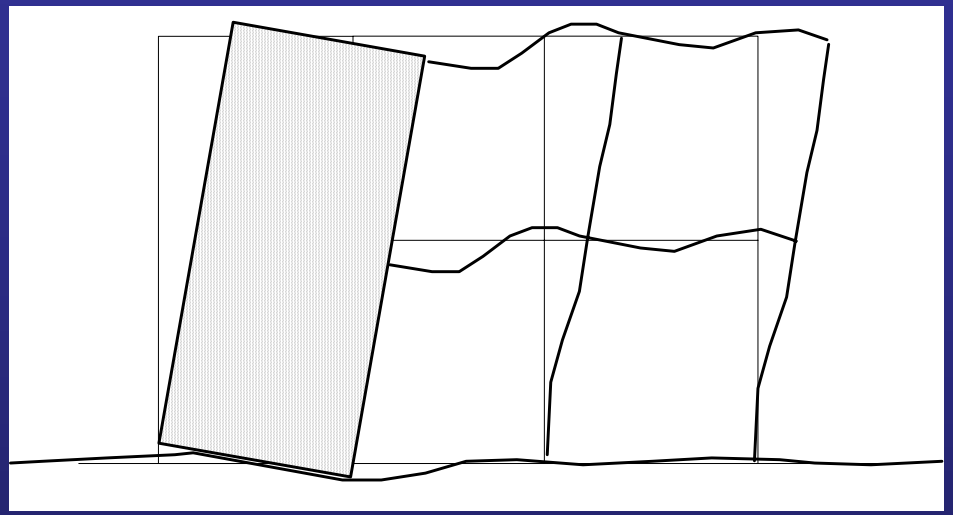


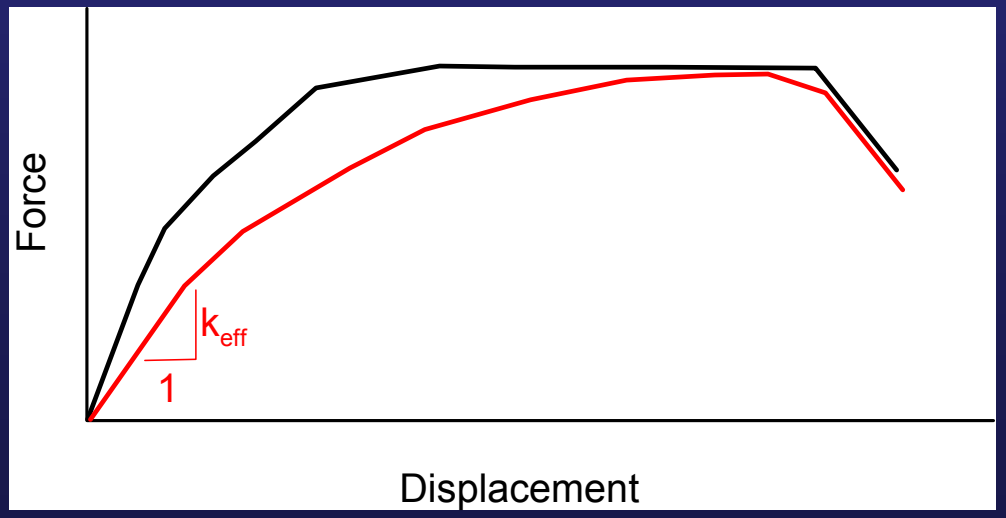


# C. Inertial Interaction. Typical Application

5. Evaluate $\tilde{T}/T$ using structure-specific model :
   - Fixed-base period $T$
   - Flexible-base period $\tilde{T}$
   - Calculate ratio $\tilde{T}/T$
   - Ductility correction:

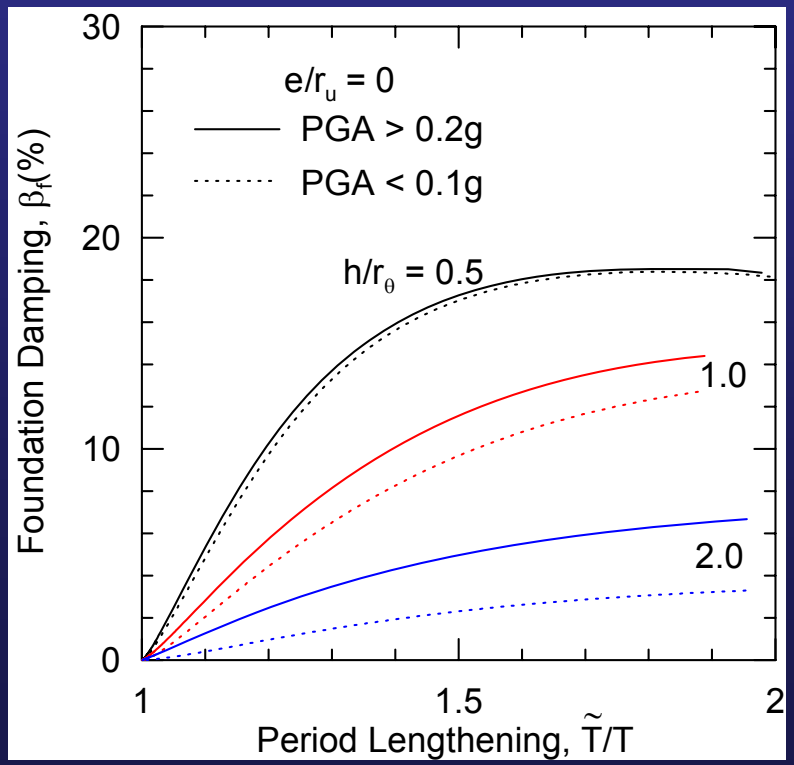
$$\frac{\tilde{T}_{eff}}{T_{eff}} = \left\{1 + \left(\frac{\mu_f}{\mu_s}\right)\left[\left(\frac{\tilde{T}}{T}\right)^2 - 1\right]\right\}^{0.5}$$

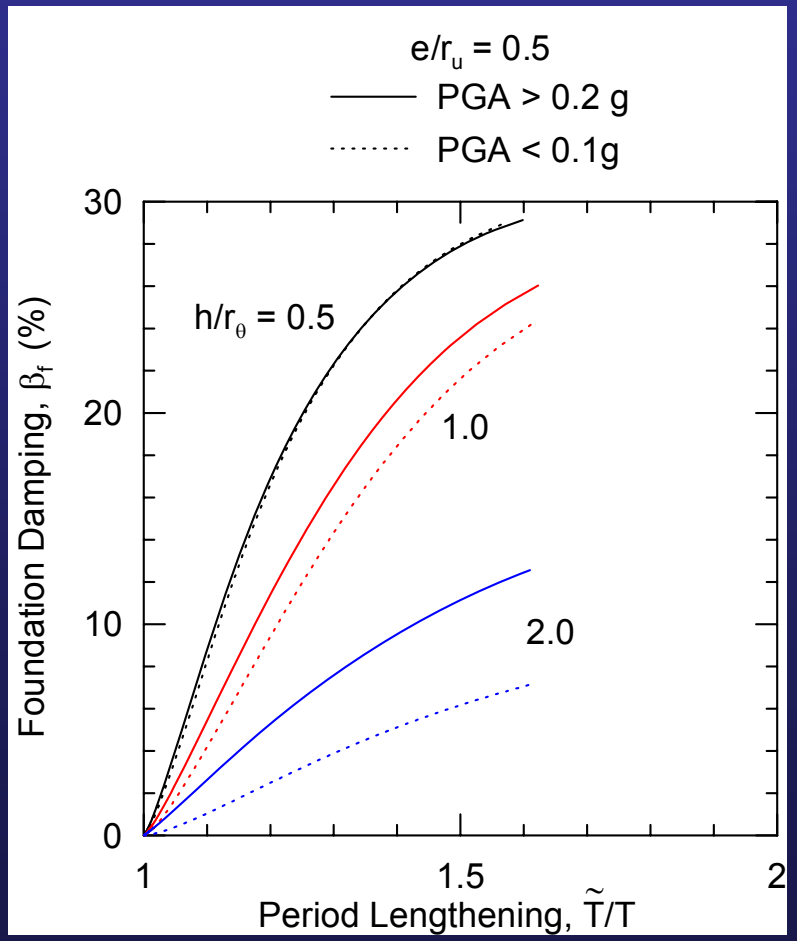




# C. Inertial Interaction. Typical application

6. Evaluate foundation damping $\beta_f$ based on $\tilde{T}_{eff}/T_{eff}$, $h/r_\theta$ and $e/r_u$



# C. Inertial Interaction. Typical application

7. Evaluate flexible-base damping ratio, $\beta_0$

$$\beta_0 = \beta_f + \frac{\beta_i}{\left(\tilde{T}_{eff} / T_{eff}\right)^3}$$

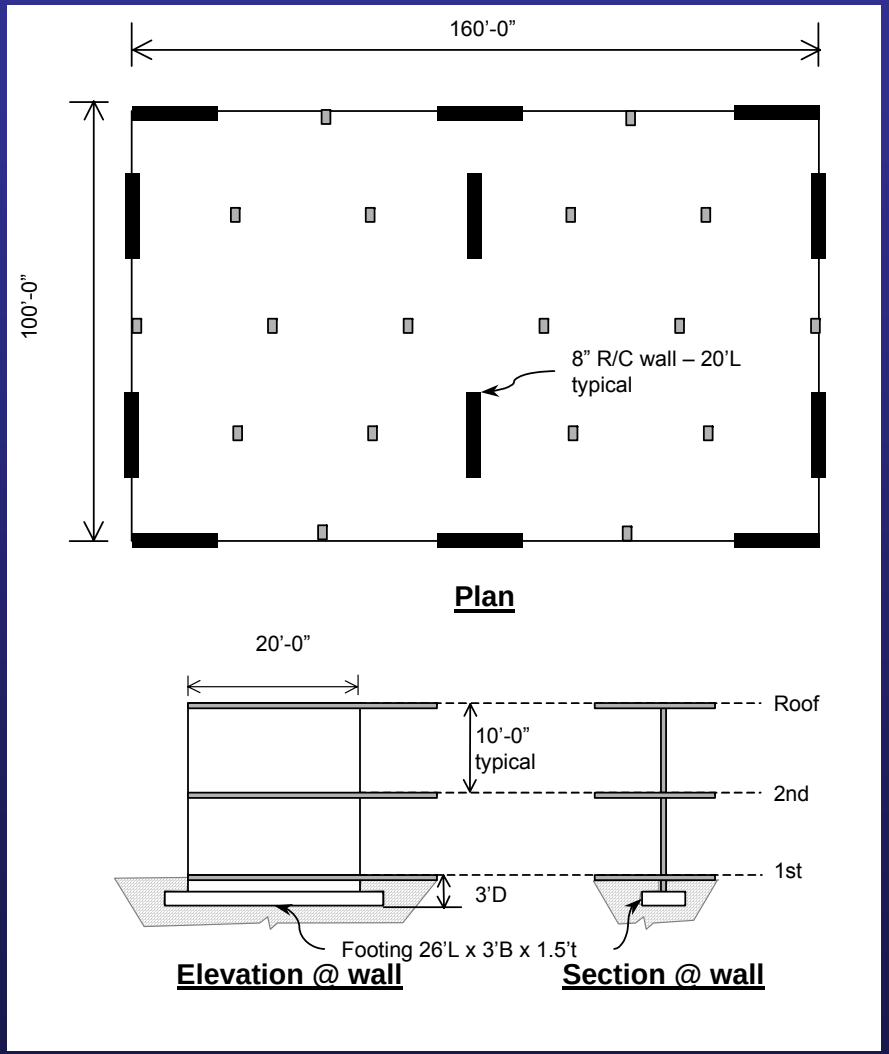
8. Evaluate the effect on spectral ordinates of the change in damping from $\beta_i$ to $\beta_0$

$$\tilde{S}_a = S_a \left( \frac{C_1 - C_2 \ln(100\beta_0)}{C_3} \right)$$

Eq. 3-7 and 3-8 of FEMA440 (assumes $\beta_i$ = 0.05)

# Limitations

- If distributed shear walls, must consider coupling of wall rotations

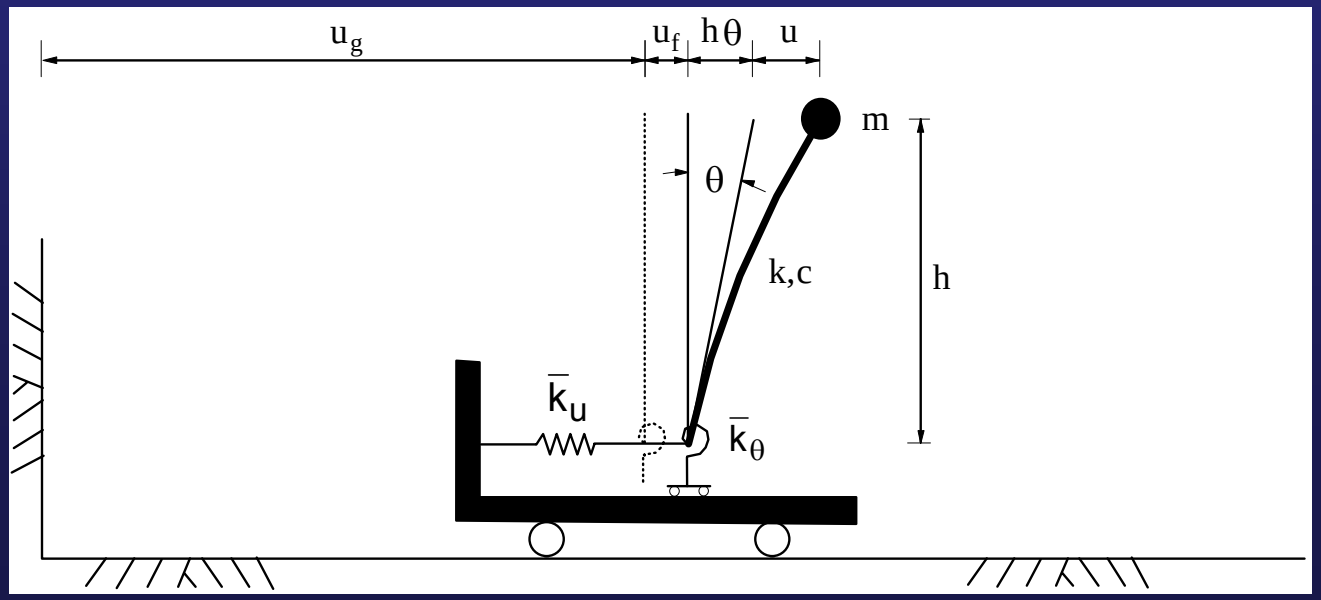


# Limitations

- If distributed shear walls, must consider coupling of wall rotations

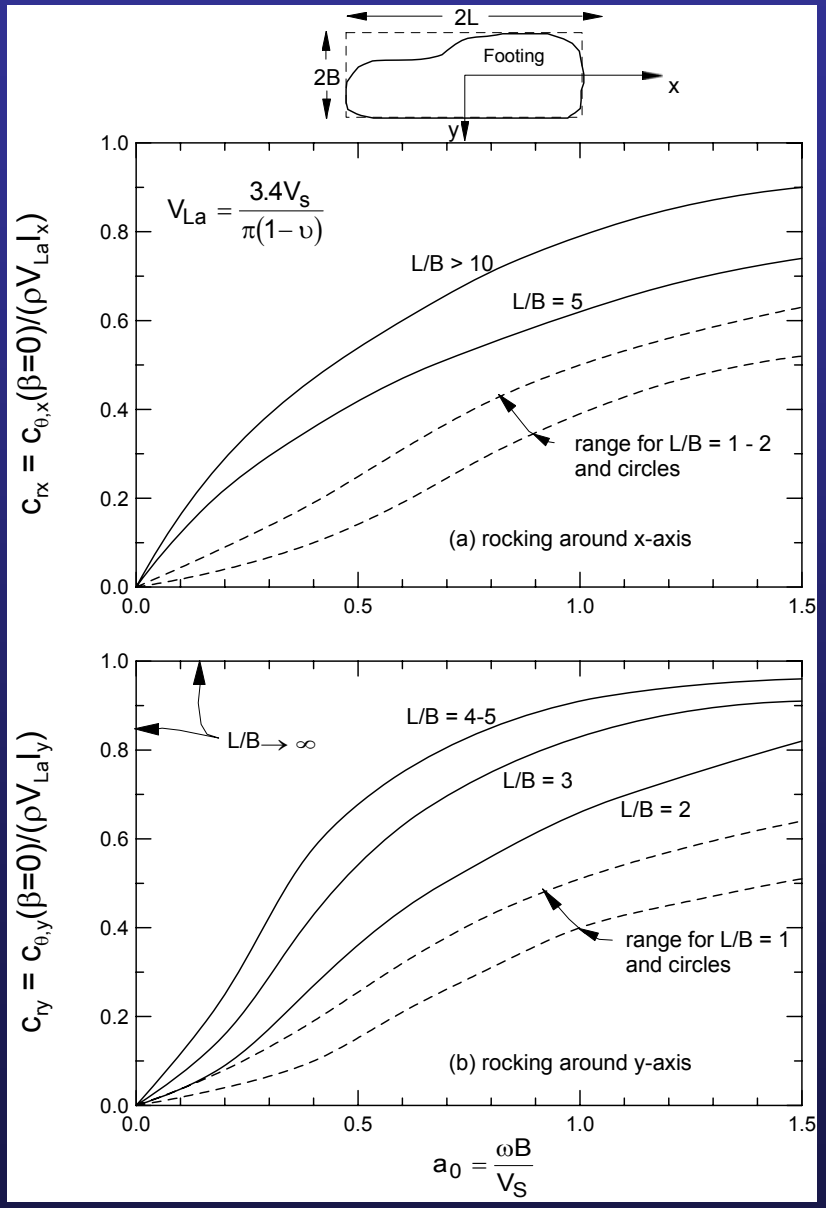
- Evaluate $k$
- Evaluate $k_u$
- Derive $k_\theta$
- Derive $r_\theta$ from $k_\theta$

$$\frac{\tilde{T}}{T} = \sqrt{1 + \frac{k}{k_u} + \frac{kh^2}{k_\theta}}$$



# Limitations

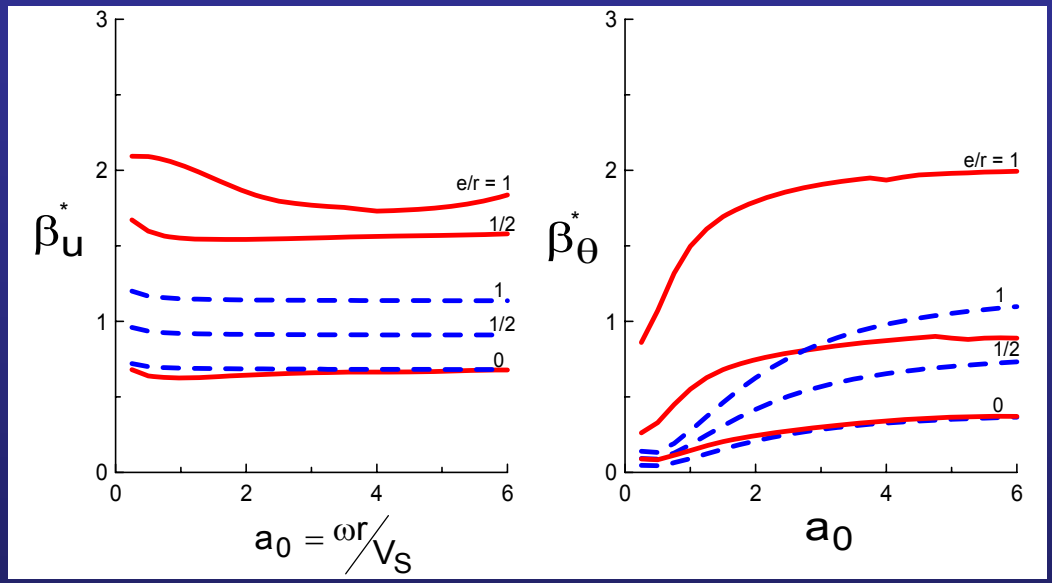
- If distributed shear walls, must consider coupling of wall rotations
- Analysis is conservative for:
  - High foundation aspect ratios (*a/b* > 2)




Modified from: Dobry and Gazetas, 1986

# Limitations

- If distributed shear walls, must consider coupling of wall rotations
- Analysis is conservative for:
  - High foundation aspect ratios (*a/b* > 2)
  - Deeply embedded foundations ($e/r_u > 0.5$)



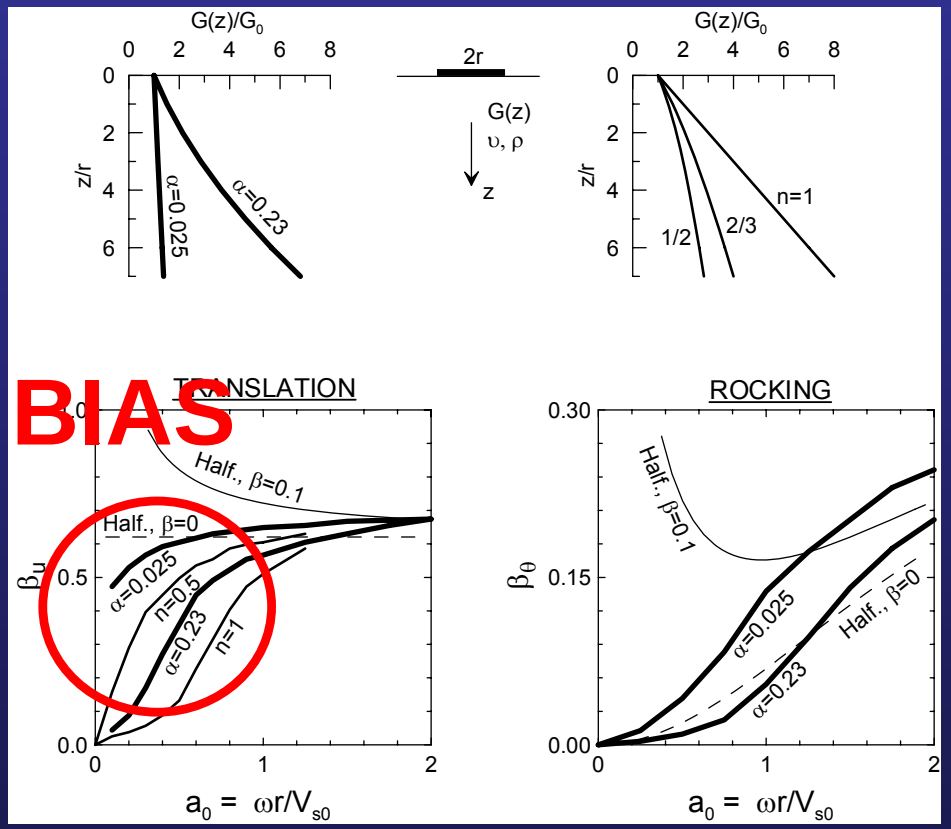

$$c_u = \beta_u \frac{K_u r_u}{V_s}$$

$$c_\theta = \beta_\theta \frac{K_\theta r_\theta}{V_s}$$

Modified from: Apsel and Luco, 1987

# Limitations

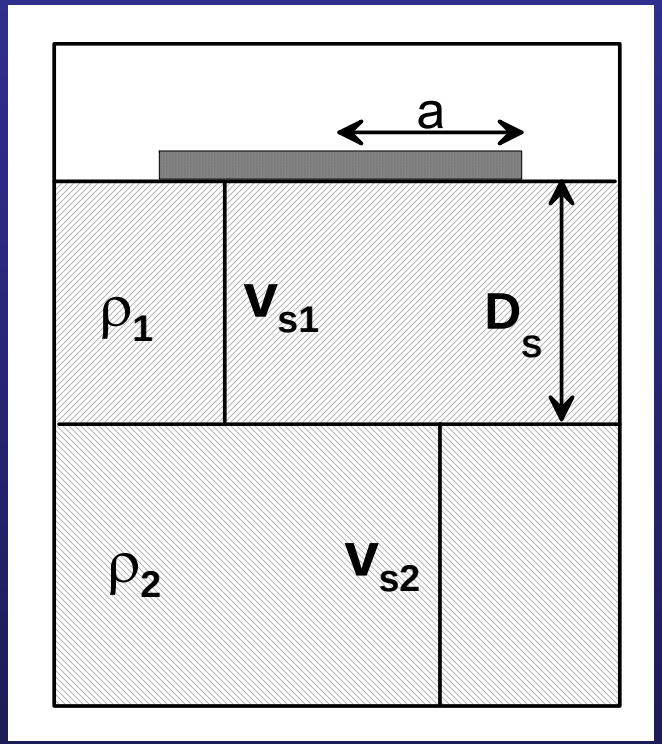
- If distributed shear walls, must consider coupling of wall rotations
- Analysis is conservative for:
  - High foundation aspect ratios (*a/b* > 2)
  - Deeply embedded foundations ($e/r_u$ > 0.5)
- Analysis unconservative for:
  - nonuniform profiles, $a_0$<1




after Gazetas, 1991

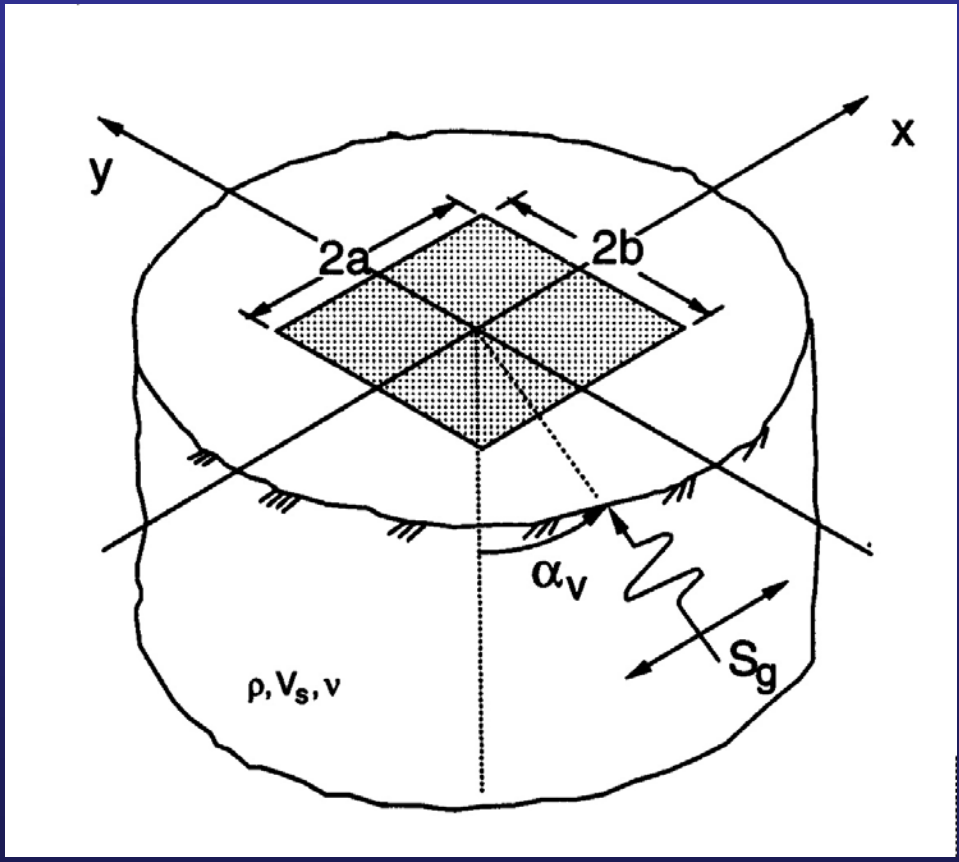
# Limitations

- If distributed shear walls, must consider coupling of wall rotations
- Analysis is conservative for:
  - High foundation aspect ratios ($a/b > 2$)
  - Deeply embedded foundations ($e/r_u > 0.5$)
- Analysis unconservative for:
  - nonuniform profiles, $a_0<1$
  - large impedance contrast at depth; $V_{s2} \geq 2 \times V_{s1}$



# D. Kinematic Interaction

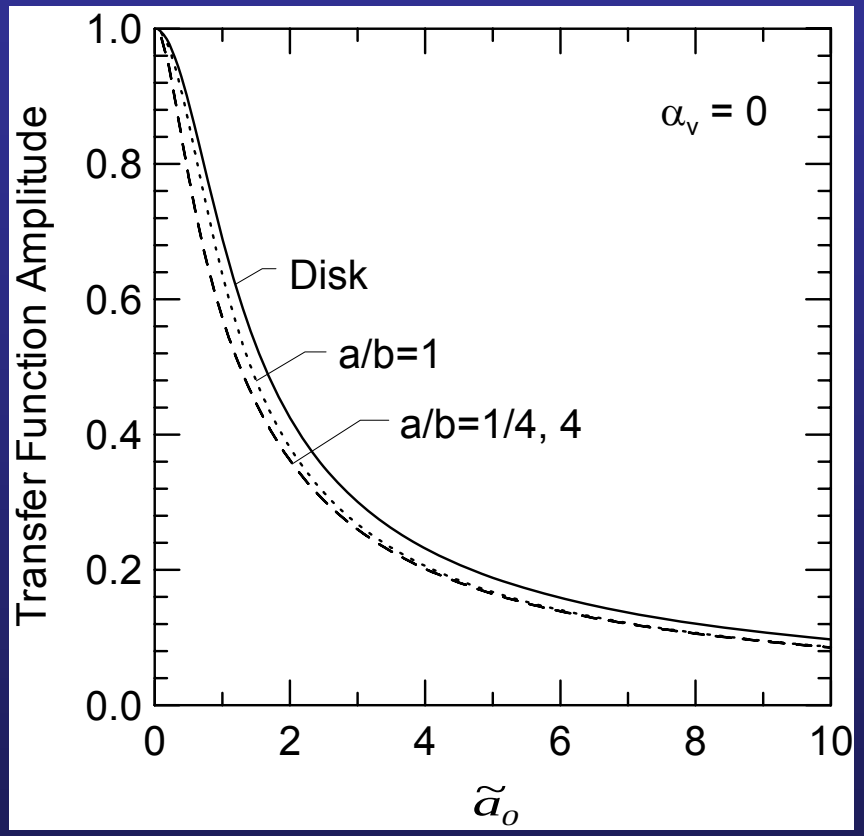
- Contributions from:
  - Base-slab averaging
  - Foundation embedment




after Veletsos et al., 1997

# D. Kinematic Interaction. Base Slab Averaging

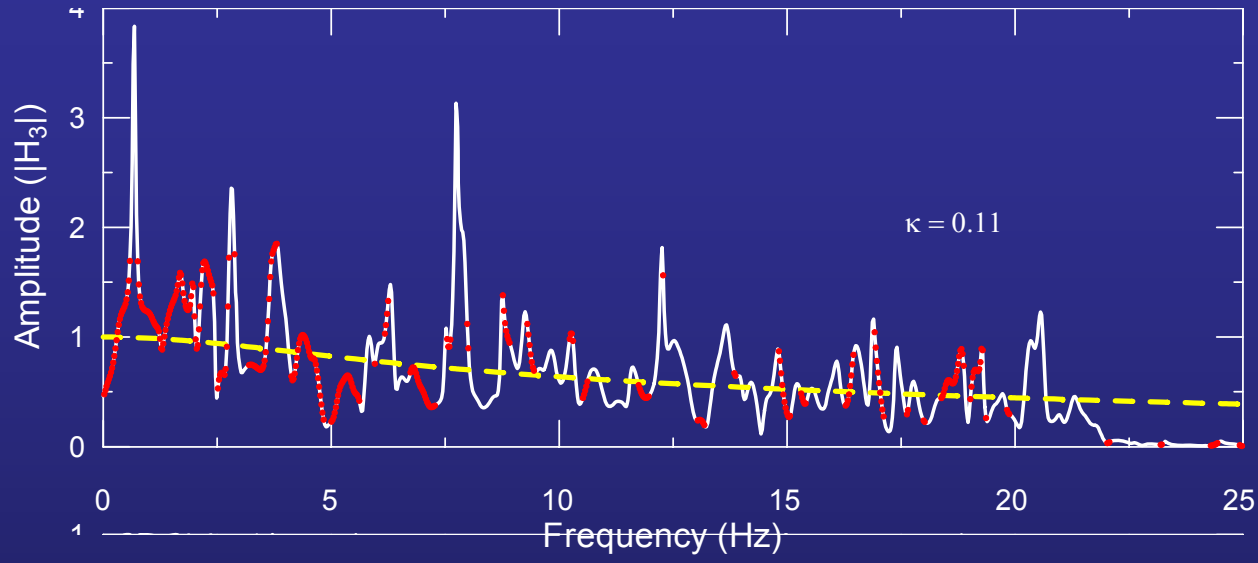
- Existing theoretical models
  - User-specified incoherence parameter, κ
  - Rigid foundation, soil is uniform halfspace
- Result is foundation / free-field transfer function, not RRS

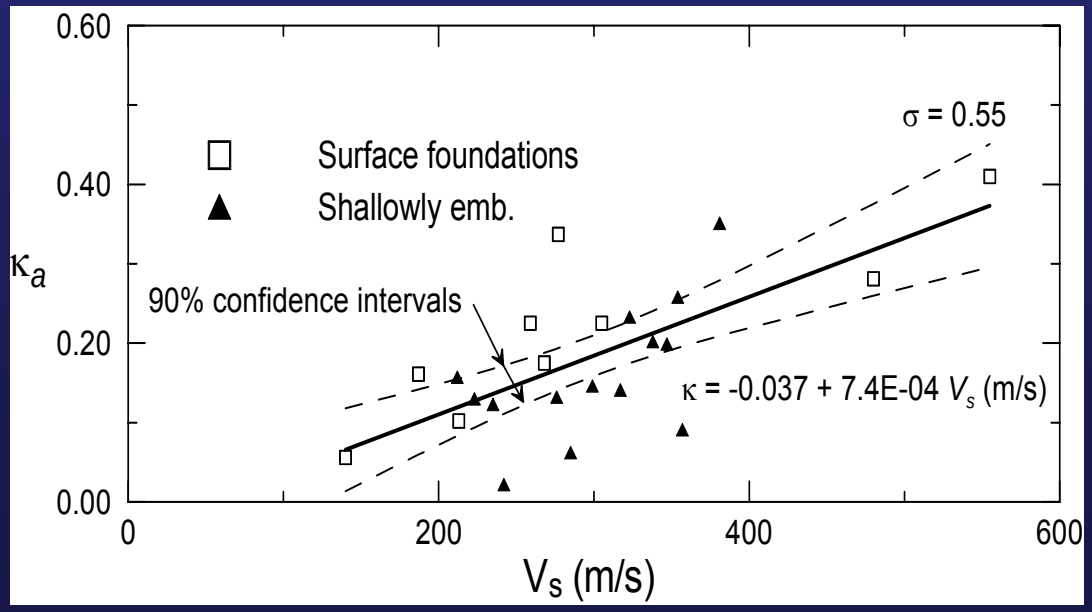



$$\tilde{a}_0 = \frac{\omega b_e}{V_{s,r}} \sqrt{\kappa^2 + \sin^2 \alpha_v \left( \frac{b}{b_e} \right)^2}$$

after Veletsos and Prasad, 1989; Veletsos et al., 1997

# Calibration against field data




Kim and Stewart, 2003

# Calibration against field data

$$\tilde{a}_0 = \frac{\omega b_e}{V_{s,r}} \sqrt{\kappa^2 + \sin^2 \alpha_v \left( \frac{b}{b_e} \right)^2}$$

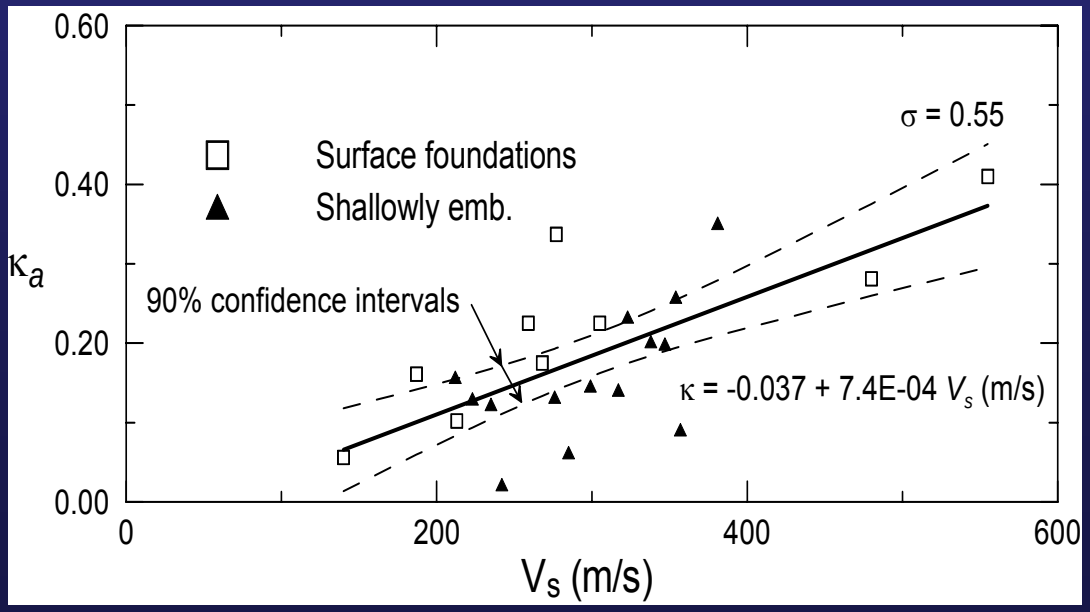
$$\tilde{a}_o = \frac{\omega b_e \kappa}{2V_{s,r}}$$

**Shear Wave Velocity Reduction Factor, $n_2$**

| | Peak Ground Acceleration, PGA (g) | | |
|---|---|---|---|
| **Site Class** | **0.1** | **0.4** | **0.8** |
| **A** | 1.00 | 1.00 | 1.00 |
| **B** | 1.00 | 0.97 | 0.95 |
| **C** | 0.97 | 0.87 | 0.77 |
| **D** | 0.95 | 0.71 | 0.32 |
| **E** | 0.77 | 0.22 | * |
| **F** | * | * | * |

Note: Use straight line interpolation for intermediate values of PGA
* = should be estimated from site-specific analysis

$$\tilde{a}_0 = \frac{\omega b_e \kappa}{2V_{s,r}} \approx \frac{\omega b_e n_1 V_s}{2n_2 V_s} = \frac{\omega b_e n_1}{2n_2}$$

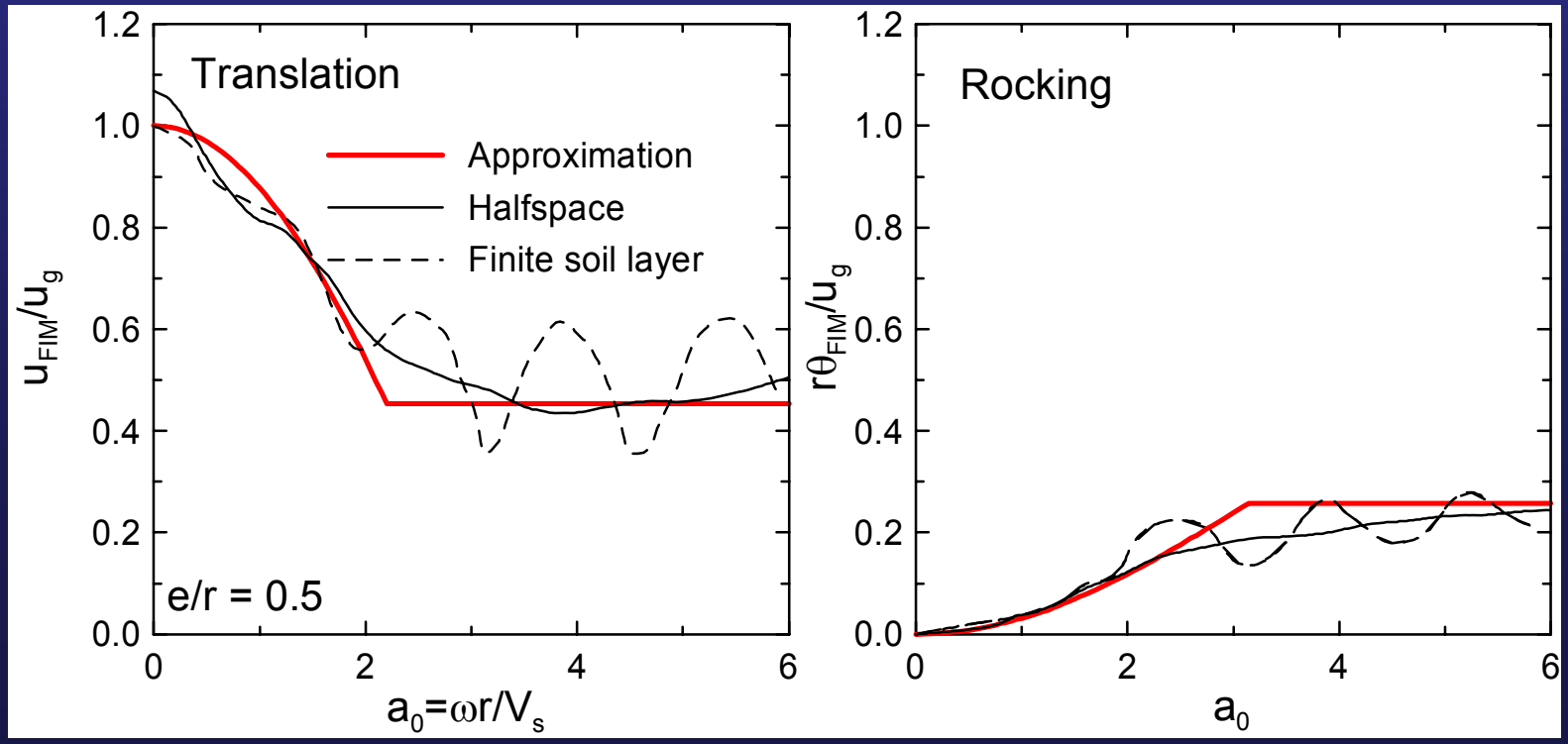



Kim and Stewart, 2003

# D. Kinematic Interaction. Embedment Effects

Elsabee and Morray (1977) and Day (1978):

- Evaluated transfer functions for vertically incident, coherent waves
- Developed simple model



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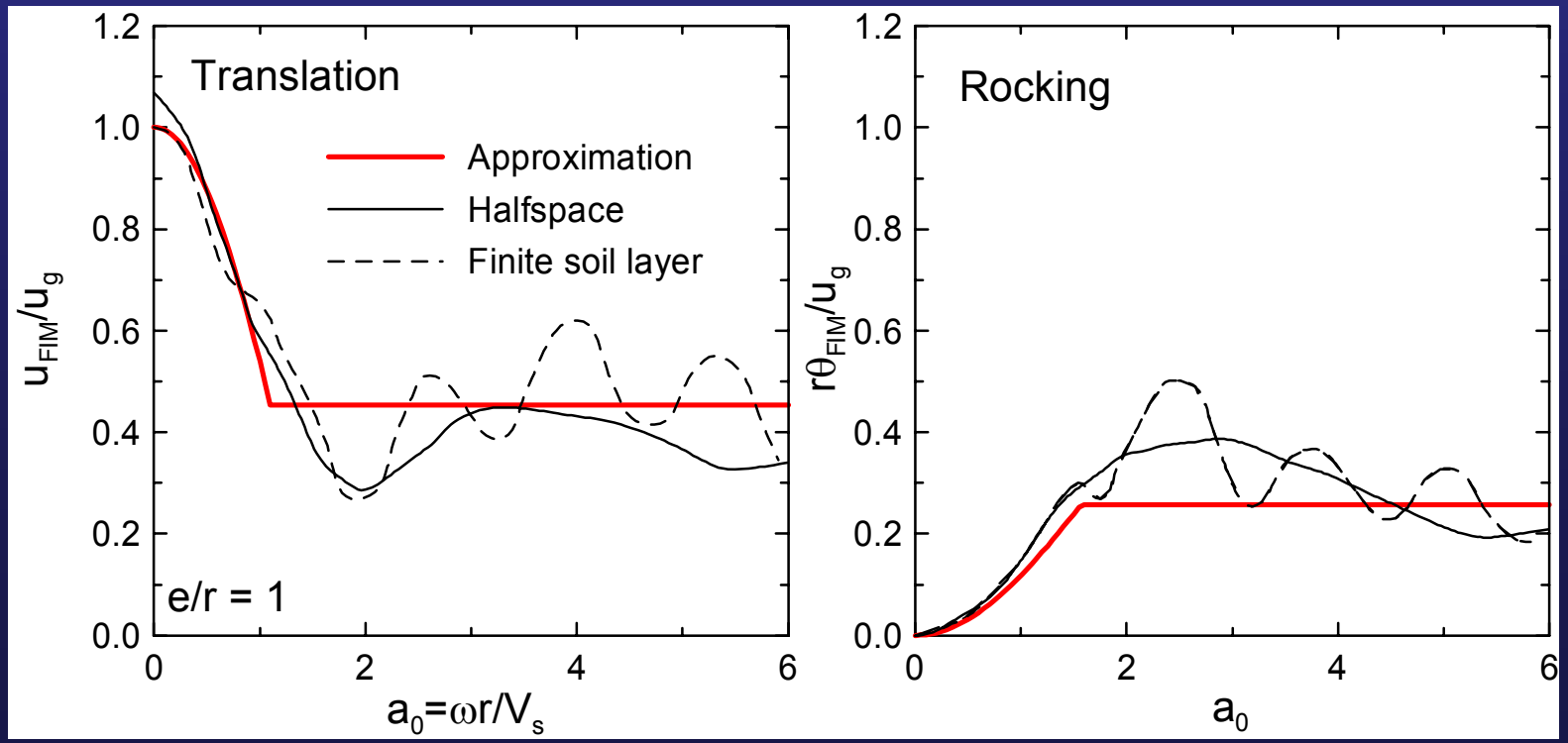
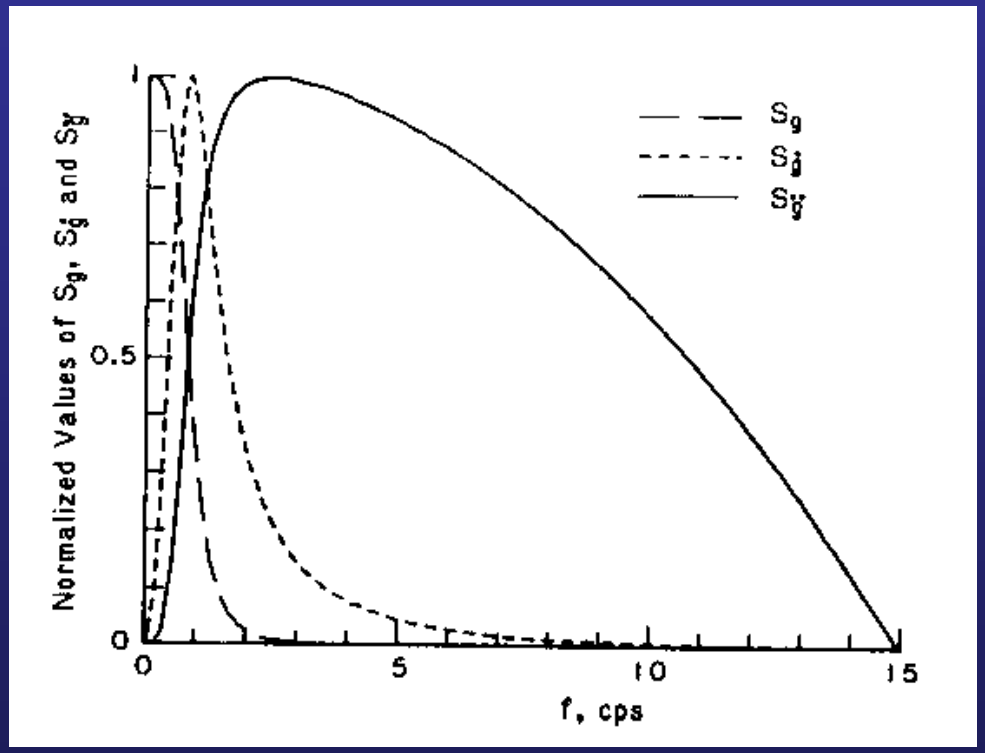


# D. Kinematic Interaction. Transfer Function to RRS

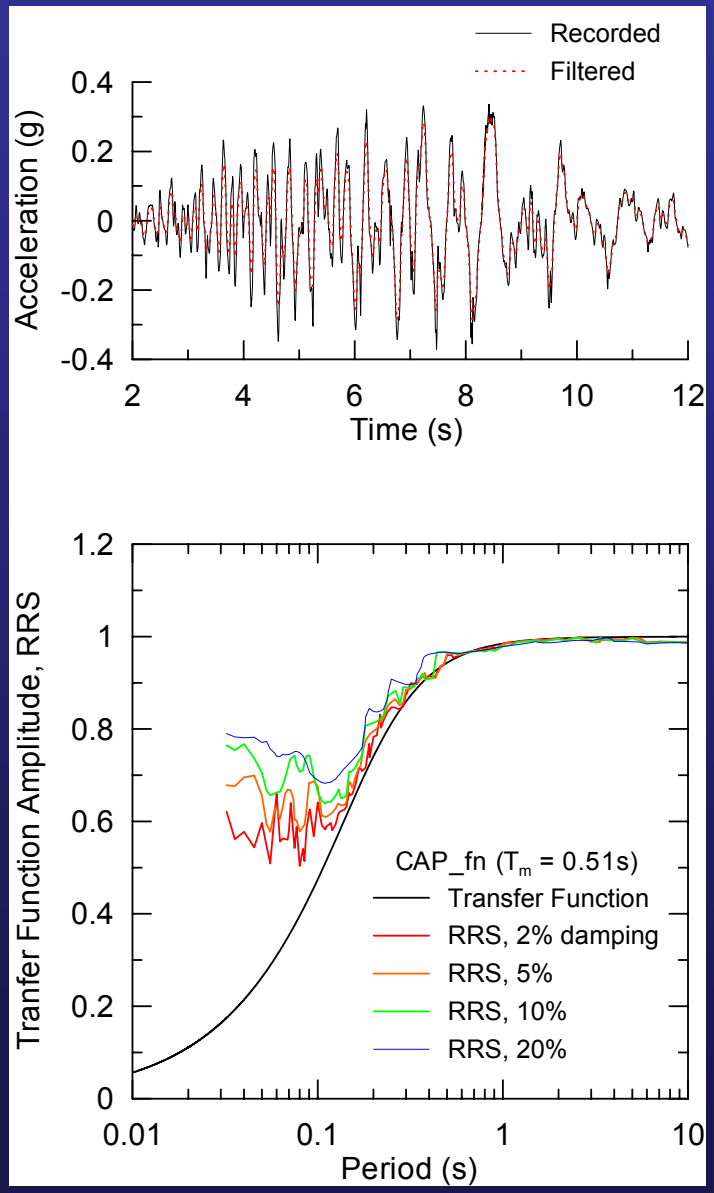
Veletsos and Prasad (1989):

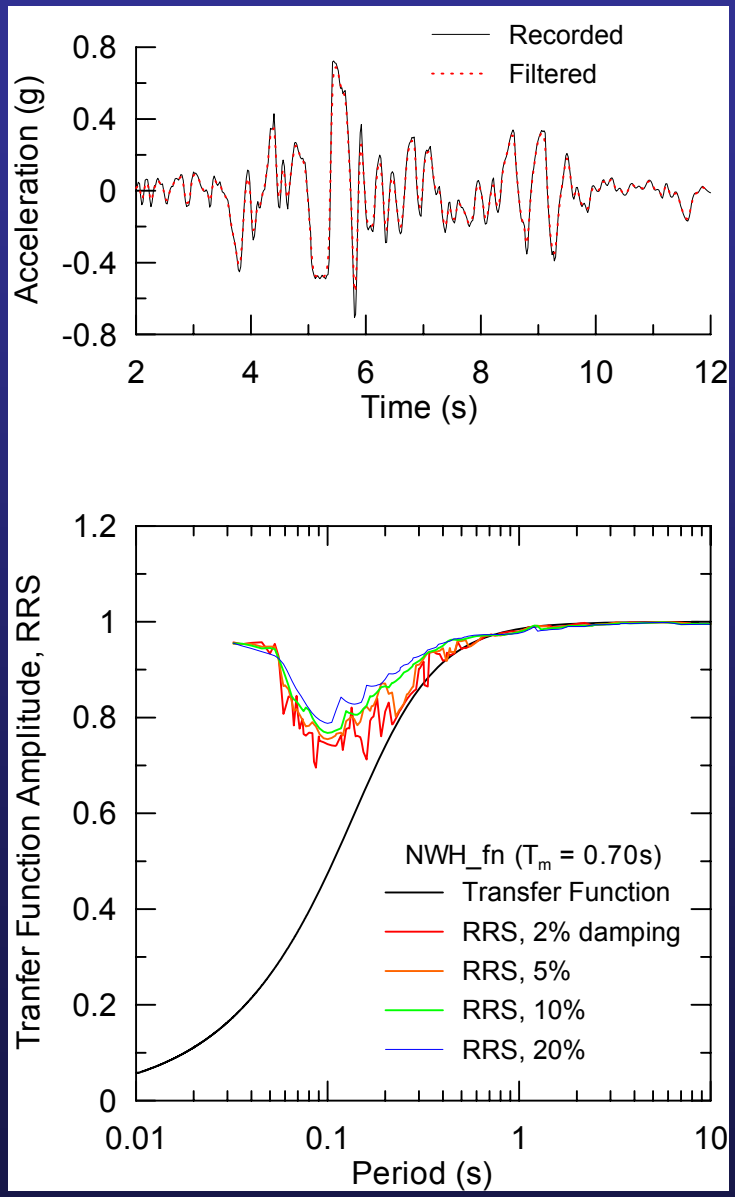
- Evaluated RRS at 2% damping for conditions where transfer function amplitude (TFA) known
- Result:
  - RRS ≈ TFA for T > 0.2 s
  - RRS ≈ TFA @ T = 0.2 s for T < 0.2 s
- Result valid for free-field spectrum shown to right




Power spectral density of ff motion
Source: Veletsos and Prasad (1989)

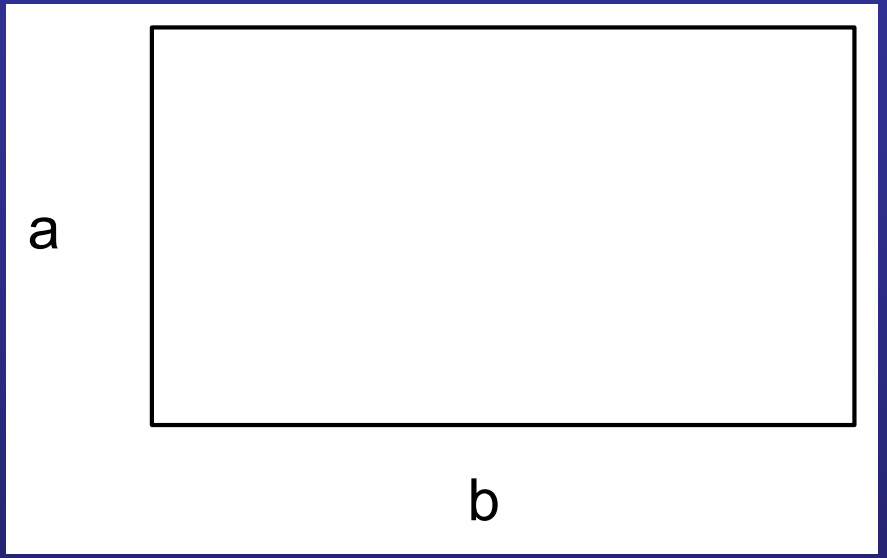
# D. Kinematic Interaction. Transfer Function to RRS



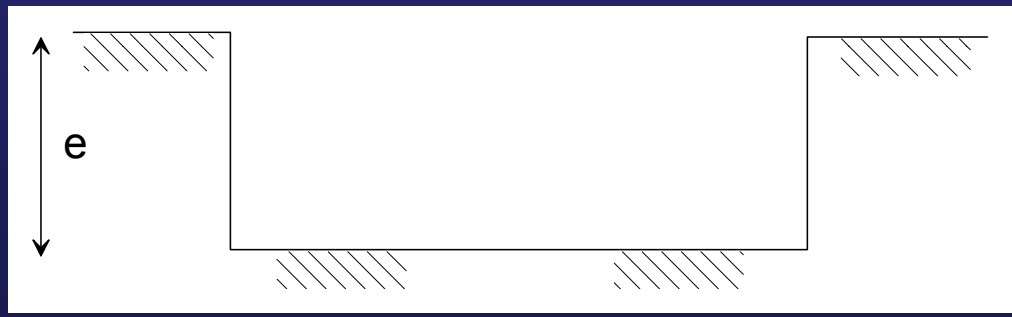



# Procedure for KI

- Evaluate effective foundation size, $b_e = \sqrt{ab}$

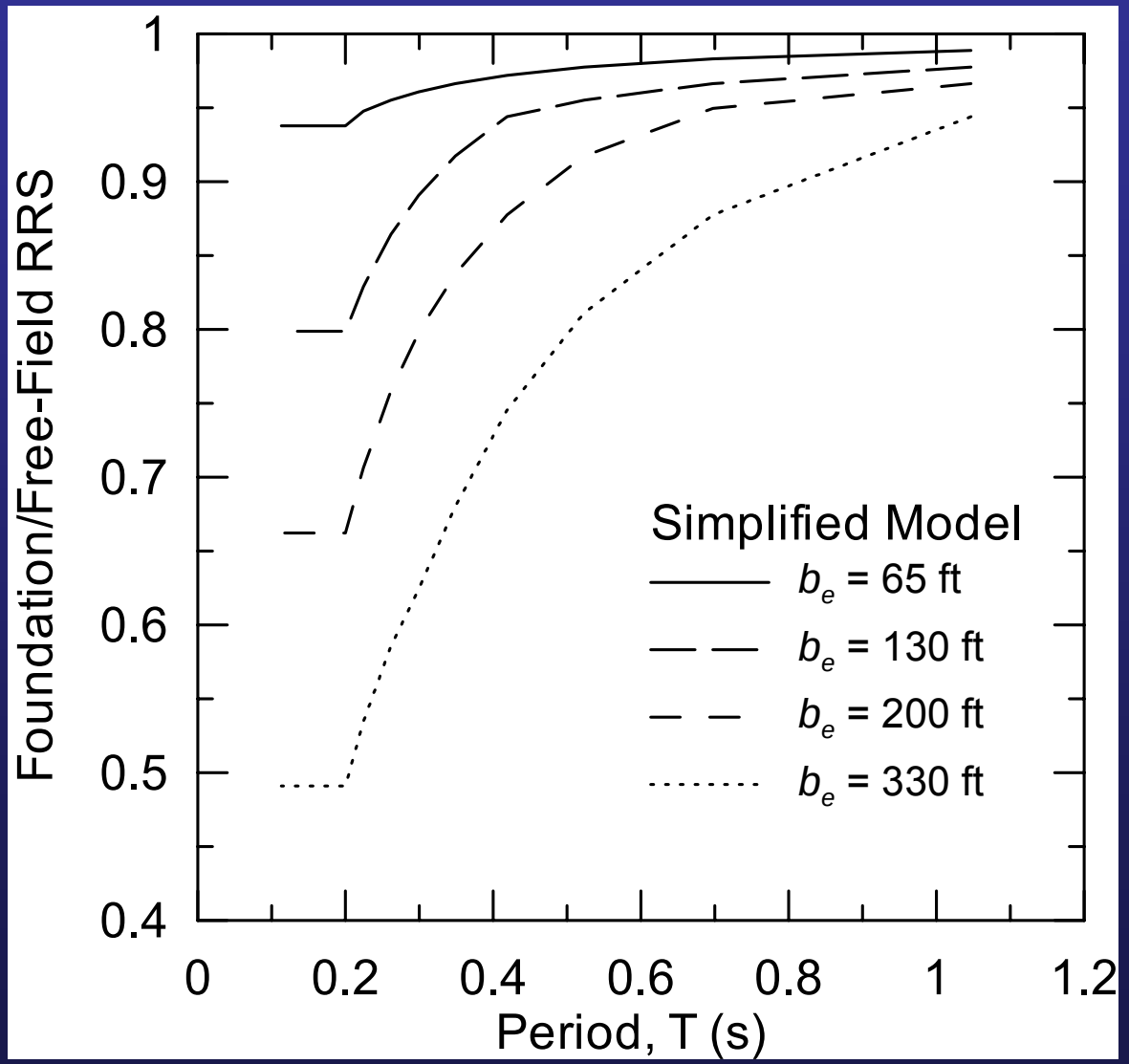



- Evaluate embedment depth, *e*



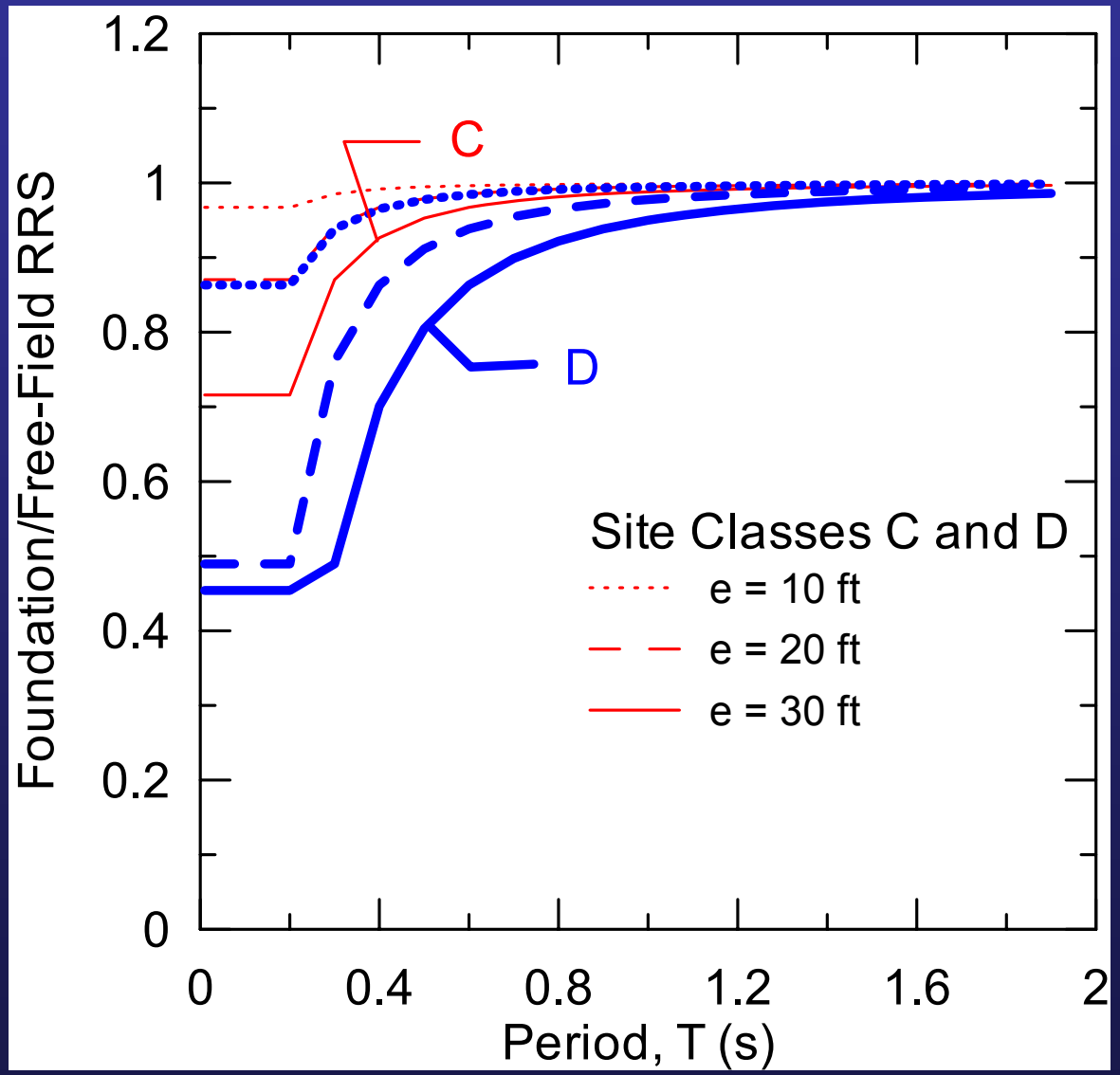
# Procedure for KI

- Evaluate RRS from base slab averaging, $RRS_{bsa}$



# Procedure for KI

- Evaluate RRS from embedment: $RRS_e$
- $RRS = RRS_{bsa} \times RRS_e$



# Limitations of KI Procedure

- Neglect KI effects for soft clay sites (NEHRP E)
- Firm rock sites (i.e., NEHRP A and B):
  - Neglect embedment effects
  - Based slab averaging model conservative (over-estimates RRS)
- Base slab averaging model not applicable for
  - Flexible foundations (non-interconnected)
  - Pile-supported foundations with slab-soil gap

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